MATH 353   Handout #0: Review

A

1. Sketch some level curves of the function $f(x, y) = x^3 - 3xy^2$. Try to sketch the graph of this function. Why do you think this function is called a monkey saddle?

B

2. Find the gradient: $T(x, y, z) = x^3 y + y^3 z + z^3 x$.

3. Let $f(x, y) = x^3/(x^2 + y^2)$ when $(x, y) \neq (0, 0)$ and $f(x, y) = 0$ otherwise. Calculate the partial derivatives $f_x(0, 0)$, $f_y(0, 0)$, $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ or explain why they do not exist.

4. Find the gradient and second-order partial derivatives of $u(x, y) = k(\ln(\cos(x/k)) - \ln(\cos(y/k))$ where k is any non-zero constant.

5. Find the gradient and second-order partial derivatives of $f(x, y) = 2xy^2 - x^2 y + 4xy$.

C

6. Determine whether or not the sequence converges. If it does, find the limit.

   (a) $a_n = \frac{(-1)^n e^n}{n!}$

   (b) $a_n = \frac{\ln(n)}{\arctan n}$

   (c) $a_n = \frac{n^{100} + 2^n \pi}{2^n}$