

**MATH 353      Handout #1**

**A**

1. Find the boundary  $\partial S$ . Is the set  $S$  closed, open, bounded?

(a)  $S = \left\{ (x, y); \frac{|x|}{|y|} \leq 1 \right\}$ .

(b)  $S = \{(x, y); y - 2x = 1, 1 \leq y \leq 3\}$

(c)  $S = \{\text{all irrational numbers between 0 and 1}\} \subset \mathbb{R}$

2. Classify all critical points of the function  $f(x, y) = xye^{-2x^2 - \frac{y^4}{4}}$ .

**B**

3. Find the boundary  $\partial S$ . Is the set  $S$  closed, open, bounded?

(a)  $S = \{(x, y); \ln(xy) \leq 0\}$ .

(b)  $S = \{(x, y); 0 < x^2 + y^2 < 4\}$

(c)  $S = \left\{ \frac{n}{3n+1} \right\}_{n=1}^{\infty} \subset \mathbb{R}$ .

4. Classify all critical points of the function  $f(x, y) = 2xy^2 - x^2y + 4xy$ .

**C**

5. Find the boundary  $\partial S$ . Is the set  $S$  closed, open, bounded?

(a)  $S = \left\{ (x, y); \frac{x^2}{y} \geq 1 \right\}$ . Sketch the set in the  $xy$ -plane.

(b)  $S = \{(x, y, z); x^2 + y^2 + 2z^2 = 4\}$ .

(a) Find all local extrema of the function  $f(x, y) = xy(4 - x - 4y)$ ;

(b) Find the absolute max/min values of  $f$  on the triangle  $\triangle ABC$  with vertices  $A(0, 0)$ ,  $B(0, 1)$  and  $C(1, 0)$ .

**D**

6. Sketch the set  $S$ . Find the boundary  $\partial S$ . Is the set  $S$  closed, open, bounded?

(a)  $S = \{(x, y); y \leq \frac{1}{x}\}$

(b)  $S = \{(x, y); 9 < \frac{1}{x^2+y^2}\}$

$$S = \{\sqrt[n]{n}\}_{n=1}^{\infty} \subset R$$

7. Find all local extrema i.e.

Classify all critical points of the function  $f(x, y) = 3y^3 - x^2y + x^2$ .