MATH 353
Midterm Review

1. Let $S = \{(x, y) \mid \sqrt{y-x} < 3\}$.
   (a) Find the boundary ∂S.
   (b) Is the set open, closed or neither?
   (c) Is the set bounded?
   (d) Sketch the set.

2. Let $S = \{(r, \theta) | 0 < r \leq \cos(\theta),\ 0 \leq \theta \leq \pi\}$ be a set in the plane described in polar coordinates.
   (a) Is the set open, closed or neither?
   (b) Find the boundary ∂S.

3. Find the critical points of the function $f(x, y) = (x^2 + y)e^{y/2}$.

4. Find the absolute maximum and minimum values of $f(x, y) = 4xy^2 - x^2y^2 - xy^3$ on the closed triangle T in the xy-plane with vertices $(0, 0)$, $(0, 6)$ and $(6, 0)$.

5. Find the absolute minimum of $f(x, y) = 1 - \sqrt{1 - x^2 - y^2}$ on the triangle T with vertices $(-1, 0)$, $(1/2, 1/2)$ and $(1/2, -1/2)$.

6. Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid $9x^2 + 36y^2 + 4z^2 = 36$.

7. Evaluate $\int\int_T \frac{\sin(y)}{1-y}\, dA$ where T is the triangular region bounded by the x-axis, the y-axis and the line $y = 1 - x$, if it is convergent.

8. Evaluate $\int\int_S \frac{1}{x^2+y^2}\, dA$ where S is the region in the first quadrant bounded by the x-axis, the y-axis and the circle $x^2 + y^2 = 1$, if it is convergent.

9. Evaluate $\iint_D \frac{1}{(x^2+y^2)^{n/2}} dA$ where n is an integer and D is the region bounded by the circles centered at the origin of radii r and R where $0 < r < R$. For what values of n does the integral have a limit as $r \to 0^+$?

10. Evaluate the integral

$$\int\int\int_R z\ dV$$

over the region R between the paraboloids $z = x^2+y^2$ and $z = 4-x^2-y^2$ using cylindrical or spherical coordinates.

11. Find the volume of the solid that lies within both the cylinder $x^2+y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

12. Evaluate $\iiint_E x^2 dV$, where E is bounded by the xz-plane and the hemispheres $y = \sqrt{9 - x^2 - z^2}$ and $y = \sqrt{16 - x^2 - z^2}$.

13. Use the transformation $u = x - y$, $v = x + y$ to evaluate $\iint_R \frac{x-y}{x+y} dA$, where R is the square with vertices $(0, 2)$, $(1, 1)$, $(2, 2)$ and $(1, 3)$.