MATH 353 FINAL HANDOUT

1. For $f(x,y) = x^2 (y+1)^3 + y^2$show that $(0,0)$ is a local minimum, then decide if it is also absolute min.Explain.

2. Sketch the region of integration and evaluate

$$\int_{-1}^{0} \left(\int_{-1}^{\sqrt[3]{y}} \frac{dx}{x^4+1} \right) dy.$$

3. Find all points on the sphere $x^2 + y^2 + z^2 = 36$ closest to $P(1,2,2)$.

4. Express the integral

$$\iint_D \frac{x+y}{x^2+y^2} dxdy$$

where D is the region above the line $x + y = 2$ and inside the circle $x^2 + y^2 = 4$

   (a) as interated integrals in cartesian coordinates;
   (b) as iterated integrals in polar coordinates

then evaluate only once.

5. Evaluate $\iiint_B \frac{zdV}{\sqrt{x^2+y^2}}$where $B = \{(x,y,z)\,; 1 \leq z \leq 4 \text{ and } z \geq x^2 + y^2\}$.

6. Find the surface area of S where S is the part of $z = \sqrt{3x^2 + 3y^2}$ below the plane $x + z = 4$.

7. Find $\oint_c \mathbf{F} \cdot \mathbf{ds}$

where $\mathbf{F} = (y^3 x + \cos(x^2), e^{y^2} + \sin(\pi x))$ and c is boundary of the triangle T from $(0,2)$ to $(2,2)$ to $(2,0)$ and back to $(0,2)$.

8. Show that for any smooth vector field $\mathbf{F}(x,y)$ and any smooth real-valued function $\phi(x,y)$

$div\,(\phi \mathbf{F}) = grad\phi \bullet \mathbf{F} + \phi div\ \mathbf{F} \qquad \boldsymbol{\nabla} \cdot (\phi \mathbf{F}) = \boldsymbol{\nabla} \boldsymbol{\phi} \cdot \mathbf{F} + \phi (\boldsymbol{\nabla} \cdot \mathbf{F})$.

9. Evaluate $\oint_c \mathbf{F} \cdot \mathbf{ds}$ where $\mathbf{F} = (x^2 + y, y^3 - x, z^4)$

and c is given as $\{x^2 + y^2 = 4\} \cap \{2x - 3y + z = 2\}$ oriented positively.

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10. Find the flux of $\mathbf{F} = (x^2, y^2, z^2)$ outward from the closed surface $S = \{x^2 + y^2 + 4(z-1)^2 = 4\}$.

11. Evaluate $\int_c \mathbf{F} \bullet \mathbf{ds}$ where $\mathbf{F} = (y, x, 2z)$ and $c = \{z = 2xy\} \cap \{x^2 + y^2 = 2\}$ from $A\,(-1, 1, -2)$ to $B\,(1, 1, 2)$.

12. For the vector field $\mathbf{F}\,(x, y, z) = (\arctan z(x^2 + y^2)\,, \ln(1 + y^2 + z^2), y\ e^{xyz})$ find *div* $\mathbf{F}$ and *curl* $\mathbf{F}$ in the domain.