MATH 353
Handout #5

1. Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 2$ that lies inside the paraboloid $z = x^2 + y^2$.

2. Find the flux of $\mathbf{F} = \mathbf{i} + \mathbf{j} + z(x^2 + y^2)^2\mathbf{k}$ out of the surface $S = \partial B$ the boundary of the solid $B = \{(x, y, z)\,; x^2 + y^2 \leq 4, 0 \leq z \leq 3\}$.

   (cylinder including top and bottom)

3. Find the surface area of S

   (a) which is the part of the cylinder $x^2 + y^2 = 4$ in the first octant below the plane $2x + y + z = 5$;

   (b) which is the part of the plane $2x + y + z = 5$ inside the cylinder $x^2 + y^2 = 4$.

4. Find the flux $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where S is the part of the cylinder $y^2 + z^2 = 4$ which lies inside the cylinder $x^2 + y^2 = 4$,above the xy-plane,oriented upward, and the field is $\mathbf{F}(x, y, z) = (x^2yz, y, xz)$.

5. Evaluate $\int\int_S zx\ dS$ where S is the part of $z = \frac{x^2}{2}$ which lies inside $x^2 + y^2 = 1, x > 0, y < 0$.

6. Evaluate $\iint_S x^2 dS$ where S is the part of the plane $x + y + z = 2$ inside the elliptical cylinder $x^2 + 2y^2 = 1$.

7. For the vector field $\mathbf{F}(x, y, z) = (xz, 4xyz^2, 2z)$ calculate $div\mathbf{F}$ and **curl F** .

8. Find all constatns A and B such that the function $f(x, y) = Ax^2y + By^3$is harmonic.