

THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS & STATISTICS
MATHEMATICS 353-02
QUIZ #1T

WINTER 2008

NAME: _____ **I.D. No.:** _____

1. Find ∂D - boundary of D . Is D closed? Open? Bounded?

Sketch/Describe the set.

(a) $D = \{(x, y) \mid y < \sqrt{x}\}$

(b) $D = \{(x, y, z) \mid 3x + 2y - z = 1\}$

(c) $D = \{x; |x| < 1\}$.

[5]

2. Find all local extrema of $f(x, y) = x^3y - 12x^2 - y^2$ in its domain. Explain.

[5]

SOLUTION

for 1a)

it must $x \geq 0$ and y is (pos, zero or negative) below the half-parabola $y = \sqrt{x}$

so the set **UNBDD**

and the boundary is $\partial D = \{y = \sqrt{x}, x \geq 0\} \cup \{x = 0, y < 0\}$

first part is out, second part is in, so **neither open nor closed**.

for 1b)

the set is a plane so $D = \partial D$ is **CLOSED and unbounded**.

for 1c)

$D = (-1, 1)$ $\partial D = \{-1, 1\}$ two points, **open and bounded**.

For 2)

f is defined, continuous, differentiable everywhere, for critical points solve

$$f_x = 3x^2y - 24x = 3x(xy - 8) = 0 \quad x = 0 \text{ OR } xy = 8$$

$$f_y = x^3 - 2y = 0 \quad x^3 = 2y$$

together if $x = 0$ then $y = 0$

$$\text{if } x \neq 0 \quad y = \frac{8}{x} \text{ and } x^3 = \frac{16}{x} \text{ then } x^4 = 16 \text{ so } x = \pm 2, y = \pm 4$$

we got 3 critical points $(0, 0), (2, 4), (-2, -4)$

for Second Derivative Test

$$f_{xx} = 6xy - 24 \quad f_{xy} = 3x^2 \quad f_{yy} = -2$$

Now $A \dots B \dots C \dots D$

$(0, 0)$	-24	0	-2	<i>neg</i>	<i>loc. max</i>
$(2, 4)$	24	12	-2	<i>pos</i>	<i>saddle</i>
$(-2, -4)$	24	12	-2	<i>pos</i>	<i>saddle</i>