The University of Calgary
Department of Mathematics and Statistics
MATH 353-02 Quiz #1a Winter 2008

Name:______________________________**I.D.#:**__________________

1. Sketch/Describe the given set S.Find the boundary ∂S .

   Is the set S closed,open,bounded? Explain.

   (a) $S = \left\{(x, y); \sqrt{xy} < 3\right\}$.

   (b) $S = \{(x, y, z); z = x^2 + y^2, z \leq 1\}$

   (c) $S = \{x; x > -5\}$. [5]

2. Find all local extrema of $f(x, y) = e^x (y^2 + 2xy)$ in the domain [5]

For 1a)

$\sqrt{....}$ is defined only $xy \geq 0$ thus the set is in the first and third quadrants including both axes $x = 0$ or $y = 0$

to solve $\sqrt{xy} < 3$ square both sides

so $0 \leq xy < 9$

for $x > 0$ we get $y < \frac{9}{x}$ below the hyperbola and above or on the x-axis y=0

and for $x < 0$ we get $y > \frac{9}{x}$ above the hyperbola and below or on the x-axis

We can see that the boundary $\partial S = \{x = 0 \text{ or } y = 0 \text{ or } xy = 9\}$

both axes and hyperbola

part is out (hyperbola),part is in (axes) so the set S is **neither open nor closed.**

Also S is unbounded.

For 1b)

we can see that the set is a paraboloid in the $xyz-$ space, cut by the plane $z = 1$

Thus $\partial S = S$, the whole boundary is inside the set ,so S is **closed and bounded.**

For 1c)

the set is the interval $(-5, \infty)$;the boundary is only $\{-5\}$ and the set is **open** and **unbounded.**

For 2)

the function f is defined and differentiable everywhere

for critical points solve

$f_x = e^x (y^2 + 2xy + 2y) = ye^x (y + 2x + 2) = 0$

$f_y = e^x (2y + 2x) = 0$ thus $y = -x$

if $y = -x$ into the first equ. $\quad -xe^x(x+2) = 0$ so $x = 0$ or $x = -2$

2 critical points $\quad (0,0), (-2,2)$

$f_{xx} = ye^x(y + 2x + 4) \quad f_{xy} = e^x(2y + 2x + 2) \quad f_{yy} = 2e^x$

points	A	B	C	D
$(0,0)$	0	2	2	4
$(-2,2)$	$4e^{-2}$	$2e^{-2}$	$2e^{-2}$	$-4e^{-4}$

$(0,0)$ is a **saddle point** since the discriminant $D = B^2 - AC > 0$

$(-2,2)$ is a **loc.min** since $A > 0, D < 0$