The University of Calgary
Department of Mathematics and Statistics
MATH 353-02 Quiz #2T(10am) Winter, 2008

Name:______________________________**I.D.#:**__________________

1. Find the absolute extrema of $f(x,y,z) = xy + z^2$ on $D = \{(x,y,z)\,;x^2+y^2+z^2 \le 2\}$. [5]

2. Evaluate $\iint\limits_D x\sqrt{1-y^2}dA$ where D is the region in the first quadrant between $y = x^2$ and $x = 1$. [5]

SOLUTION

For 1)

for C.P. inside solve $\nabla f = 0$

$y = 0 \qquad x = 0 \qquad 2z = 0 \qquad (0,0,0)$

for C.P. on theboundary solve

$\nabla f = \lambda \nabla g$ with $g(x,y,z) = x^2+y^2+z^2 = 2$

$y = \lambda 2x \qquad x = \lambda 2y \qquad 2z = \lambda 2z$

from the last equation two cases:

$1.z = 0 \qquad xy \ne 0 \qquad \lambda = \dfrac{y}{2x} = \dfrac{x}{2y} \to y = \pm x$

back to the sphere $\quad 2x^2 = 2 \qquad x = \pm 1 \qquad (\pm1,\pm1,0)\,,(\pm1,\mp1,0)$

2. $\quad z \ne 0 \qquad \lambda = 1 \qquad y = 2x$ and $x = 2y \to x = y = 0$

back to the sphere $\quad z^2 = 2 \qquad z = \pm\sqrt{2} \qquad \left(0,0,\pm\sqrt{2}\right)$

now values $\quad f(0,0,0) = 0 \qquad f(\pm1,\pm1,0) = 1$

$f((\pm1,\mp1,0)) = -1$ minimum and $f\left(0,0,\pm\sqrt{2}\right) = 2$ maxima

For 2)

the domain could be sliced vetically: $\quad 0 \le x \le 1; 0 \le y \le x^2$

OR $\quad$ horizontally $\quad 0 \le y \le 1; \sqrt{y} \le x \le 1$; it is easier to integrate w.r.to x:

$$I = \iint\limits_D x\sqrt{1-y^2}dA = \int_0^1 \sqrt{1-y^2}\left(\int_{\sqrt{y}}^1 x dx\right)dy = \int_0^1 \sqrt{1-y^2}\left(\left[\tfrac{x^2}{2}\right]_{\sqrt{y}}^1\right)dy = \tfrac{1}{2}\int_0^1 \sqrt{1-y^2}dy - \tfrac{1}{2}\int_0^1 y\sqrt{1-y^2}dy =$$

(table and subst. $u = 1-y^2, du = -2ydy$)

$$= \tfrac{1}{2}\left[\tfrac{y}{2}\sqrt{1-y^2} + \tfrac{1}{2}\arcsin y\right]_0^1 + \tfrac{1}{2}\left[\frac{(1-y^2)^{\frac{3}{2}}}{3}\right]_0^1 = \tfrac{1}{4}\arcsin 1 - \tfrac{1}{6} = \frac{\pi}{8} - \frac{1}{6}.$$

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