The University of Calgary
Department of Mathematics and Statistics
MATH 353-02 Quiz #2T(3pm) Winter, 2008

Name:____________________________**I.D.#:**__________________

1. Find the pont(s) on $x^2+y^2+z^2=36$ furthest to the point $P(1,2,2)$. [5]

2. Evaluate $\iint\limits_D y^2 e^{xy}dA$ where D is the triangle with vertices $(0,0),(0,2),(1,2)$. [5]

SOLUTION

For 1)

looking for maximum of the distance $d=\sqrt{(x-1)^2+(y-2)^2+(z-2)^2}$

it is the same as max of $f(x,y,z)=(x-1)^2+(y-2)^2+(z-2)^2$

Solve

$\nabla f=\lambda\nabla g$ with $g(x,y,z)=x^2+y^2+z^2=36$

$2(x-1)=\lambda 2x \qquad xyz\neq 0$

$2(y-2)=\lambda 2y \qquad$ so $\lambda=\dfrac{x-1}{x}=\dfrac{y-2}{y}=\dfrac{z-2}{z}$

$2(z-2)=\lambda 2z$

therefore $\quad yx-y=xy-2x \quad \rightarrow y=2x$

and $\quad zy-2z=yz-2y \quad \rightarrow y=z=2x$

back to the sphere $\quad x^2+4x^2+4x^2=9x^2=36$

so $x=\pm 2$ and we have two ponts $\quad (\pm 2,\pm 4,\pm 4)$

since $f(2,4,4)=9$ and $f(-2,-4,-4)=81$ the furthest point is $(-2,-4,-4)$.

For 2)

the domain could be sliced vetically: $\quad 0\le x\le 1; 2x\le y\le 2$

OR horizontally $\quad 0\le y\le 2; 0\le x\le \dfrac{y}{2}$; it is easier to integrate w.r.to x:

$$I=\iint\limits_D y^2e^{xy}dA=\int_0^2 y^2\left(\int_0^{\frac{y}{2}} e^{xy}dx\right)dy=\int_0^2 y^2\int_0^{\frac{y}{2}}\left[\frac{e^{xy}}{y}\right]_{x=0}^{x=\frac{y}{2}}dy=\int_0^2\left(ye^{\frac{y^2}{2}}-y\right)dy$$

(subst $u=\frac{y^2}{2}, du=ydy$) $=\left[e^{\frac{y^2}{2}}\right]_0^2-\left[\frac{y^2}{2}\right]_0^2=e^2-3.$