The University of Calgary
Department of Mathematics and Statistics
MATH 353-02
Quiz #3T(10)

Winter,2008

Name:____________________________**I.D.#:**__________________

1. Set up the integral $\iint\limits_D \frac{xy}{x^2+y^2}dxdy$ where $D = \{(x,y)\,;x \geq 1, x^2+y^2 \leq 4, y \geq 0\}$as iterated integrals in both

   (a) cartesian coordinates,

   (b) and polar coordinates, and then evaluate (only once). [6]

2. Is the integral $\iint\limits_D \frac{1}{(x+y)^2}\, dA$ where $D = \{(x,y); x \geq 1; 0 \leq y \leq 2x\}$ convergent or divergent? Explain.Sketch the set. [4]

Solution
For 1a)
sketch the set;intersection of the circle and the line $x=1$ is at $y=\sqrt{3}$
so
$1 \leq x \leq 2 \qquad 0 \leq y \leq \sqrt{4-x^2}$ OR $0 \leq y \leq \sqrt{3}\ 1 \leq x \leq \sqrt{4-y^2}$

$$\iint\limits_D \frac{xy}{x^2+y^2}dxdy = \int_1^2 x\left(\int_0^{\sqrt{4-x^2}} \frac{y}{x^2+y^2}dy\right)dx = \frac{1}{2}\int_1^2 x\left[\ln(x^2+y^2)\right]_{y=0}^{y=\sqrt{4-x^2}} dx =$$

$$= \frac{1}{2}\int_1^2 x\left(\ln 4 - \ln(x^2)\right)dx = \ln 2\left[\frac{x^2}{2}\right]_1^2 - \int_1^2 x\ln x dx = \frac{3}{2}\ln 2 - \left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^2$$

$$= \frac{3}{2}\ln 2 - 2\ln 2 + 1 - \frac{1}{4} = \frac{3}{4} - \frac{1}{2}\ln 2$$

For 1b)
in polar coord. $\iint\limits_D \frac{xy}{x^2+y^2}dxdy = \iint\limits_{D^*} \frac{r^2\cos\theta\sin\theta}{r^2} rdrd\theta$

where $\quad D^* = \{0 < r \leq 2\ ,\ r \geq \frac{1}{\sin\theta}, \theta \in \left[0, \frac{\pi}{3}\right]\}$

since $x \geq 0, y \geq 0 \to \theta \in \left[0, \frac{\pi}{2}\right]$ and

$x \geq 1 \to r\cos\theta \geq 1 \qquad \frac{1}{\cos\theta} \leq r \leq 2$ necessarily

$2 \geq \frac{1}{\cos\theta} \qquad \cos\theta \geq \frac{1}{2} \qquad \theta \leq \frac{\pi}{3}$

$$I = \int_0^{\frac{\pi}{3}} \cos\theta \sin\theta (\int_{\frac{1}{\cos\theta}}^{2} r dr) d\theta = \int_0^{\frac{\pi}{3}} \cos\theta \sin\theta \left[\frac{r^2}{2} \right]_{\frac{1}{\cos\theta}}^{2} d\theta =$$

$$= \int_0^{\frac{\pi}{3}} \cos\theta \sin\theta (2 - \frac{1}{2\cos^2\theta}) d\theta = \left[\sin^2\theta + \frac{1}{2} \ln\cos\theta \right]_0^{\frac{\pi}{3}} =$$

$$= \frac{3}{4} + \frac{1}{2} \ln\frac{1}{2} \qquad \text{as above.}$$

For 2)

$$\iint_D \frac{1}{(x+y)^2}\, dA \text{ where } D = \{(x,y); x \geq 1; 0 \leq y \leq 2x\}$$

$$= \int_1^{\infty} (\int_0^{2x} \frac{1}{(x+y)^2} dy) dx = \int_1^{\infty} \left[\frac{-1}{x+y} \right]_{y=0}^{y=2x} dx =$$

$$= \int_1^{\infty} \left[\frac{-1}{3x} + \frac{1}{x} \right] dx = \frac{2}{3} \int_1^{\infty} \frac{dx}{x} = \frac{2}{3} \left[\lim_{x\to\infty} \ln x - 1 \right] = \infty$$

divergent