The University of Calgary
Department of Mathematics and Statistics
MATH 35302
Quiz #3T(15)

Winter 2008

Name:______________________________**I.D.#:** ________________

1. Set up the integral $\displaystyle\iint_D \frac{x}{y} e^{-x^2-y^2} dxdy$ where

   $D = \{(x,y)\,; y \geq -x, x^2+y^2 \leq 2, x \leq 0\}$as iterated integrals in both

   (a) cartesian coordinates,

   (b) and polar coordinates, and then evaluate (only once). [6]

2. Is the integral $\displaystyle\iint_D \frac{1}{x\,(x^2+y^2)} dA$ where $D = \{(x,y); x \geq 1; 0 \leq y \leq x\}$

   convergent or divergent? Explain.Sketch the set. [4]

Solution
For 1a)
sketch the set;intersection of the circle and the line $y = -x$ is
in the second quadrant at $x = -1$, $y = 1$ so
$-1 \leq x \leq 0 \qquad -x \leq y \leq \sqrt{2-x^2}$

$$\iint_D \frac{x}{y} e^{-x^2-y^2} dxdy = \int_{-1}^{0} x e^{-x^2} \left(\int_{-x}^{\sqrt{2-x^2}} \frac{e^{-y^2}}{y} dy \right) dx = \text{impossible}$$

For 1b)
in polar coord. $\displaystyle\iint_D \frac{x}{y} e^{-x^2-y^2} dxdy = \iint_{D^*} \frac{r\cos\theta}{r\sin\theta} e^{-r^2} r dr d\theta$

where $\quad D^* = \{0 < r \leq \sqrt{2}\ , \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{4}\right]\}$

since $x < 0, y > 0$ $\theta \in \left[\frac{\pi}{2}, \pi\right]$ and $y \geq -x \qquad \sin\theta \geq -\cos\theta$
so $\tan\theta \leq -1$

$$I = \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \frac{\cos\theta}{\sin\theta} \left(\int_0^{\sqrt{2}} e^{-r^2} r dr\right) d\theta = \left[\ln\sin\theta\right]_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cdot \left[\frac{-1}{2} e^{-r^2}\right]_0^{\sqrt{2}} =$$

$$= \frac{1}{2}\left(\ln\frac{1}{\sqrt{2}}\right)(1-e^{-2}) = \frac{1}{4}(e^{-2}-1)\ln 2.$$

For 2)
$D = \{(x,y); x \geq 1; 0 \leq y \leq x\}$ is unbounded

$$\iint_D \frac{1}{x\,(x^2+y^2)} dA = \int_1^{\infty} \frac{1}{x} \left(\int_0^x \frac{1}{x^2+y^2} dy\right) dx = \int_1^{\infty} \frac{1}{x} \left[\tfrac{1}{x} \arctan \tfrac{y}{x}\right]_{y=0}^{y=x} dx =$$

$$= \arctan 1 \int_1^\infty \frac{dx}{x^2} = \frac{\pi}{4}\left[-\frac{1}{x}\right]_1^\infty = \frac{\pi}{4}.$$

also by Polar coord.

$D^* = \{(r,\theta); r \geq \frac{1}{\cos\theta}; 0 \leq \theta \leq \frac{\pi}{4}\}$ then

$$\iint_D \frac{dxdy}{x\left(x^2+y^2\right)} = \iint_{D^*} \frac{rdrd\theta}{r^3\cos\theta} = \int_0^{\frac{\pi}{4}} \frac{1}{\cos\theta}\left(\int_{\frac{1}{\cos\theta}}^\infty \frac{dr}{r^2}\right) d\theta = \int_0^{\frac{\pi}{4}} \frac{1}{\cos\theta}\left(\frac{-1}{r^2}\right)_{\frac{1}{\cos\theta}}^\infty d\theta =$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{\cos\theta}\left(0+\cos\theta\right) d\theta = \frac{\pi}{4}$$