The University of Calgary
Department of Mathematics and Statistics
MATH 353-02 Quiz #4T(10am) Winter 2008

Name:______________________________**I.D.#:**__________________

1. Find the value of k so that the vector field $\mathbf{F}(x,y) = (kxye^{x^2y} - \sin x, x^2e^{x^2y} - e^{-y})$ **i**s conservative then find a potential. [3]

2. Find $\int_c \mathbf{F} \bullet \mathbf{ds}$ where $\mathbf{F}(x,y,z) = (2x, z-5, -x)$ and c is the curve $\{\ z - 2y = 3\} \cap \{x^2 + y^2 = z\}$. [4]

3. Evaluate $\int_c x^2 z\ ds$ and c is given by $\mathbf{r}\,(t) = (t, t^3, t)$ between.$A\,(-1,-1,-1)$ and the origin. [3].

SOLUTION

For 1)

$F_1 = kxye^{x^2y} - \sin x \qquad F_2 = x^2e^{x^2y} - e^{-y}$

$(F_1)_y = kxe^{x^2y} + kx^3ye^{x^2y} = (F_2)_x = 2xe^{x^2y} + 2x^3ye^{x^2y} \longrightarrow\!\!\rightarrow k = 2$

looking for f such that

$f_x = 2xye^{x^2y} - \sin x \qquad f_y = x^2e^{x^2y} - e^{-y}$

integrate the first condition

$f = \int(2xye^{x^2y} - \sin x)dx + c(y) = e^{x^2y} + \cos x + c(y)$

differentiate w.r. to y and compare with F_2

$f_y = x^2e^{x^2y} + c'(y) = x^2e^{x^2y} - e^{-y} \qquad$ so $c'(y) = -e^{-y}$

so $c = e^{-y}$ and together

$f\,(x,y) = e^{x^2y} + \cos x + e^{-y} + const$

for 2)

we need to find a parametrization of c :

$z = 3 + 2y = x^2 + y^2 \rightarrow 4 = x^2 + (y-1)^2$

thus $x = 2\cos t, y = 1 + 2\sin t, z = 5 + 4\sin t, t \in [0, 2\pi]$

and

$\mathbf{r}(t) = (2\cos t, 1 + 2\sin t, 5 + 4\sin t) \quad \mathbf{r}'(t) = (-2\sin t, 2\cos t, 4\cos t)$

we need the field $\mathbf{F}(x,y,z)$ on $c \qquad \mathbf{F} \circ \mathbf{r} = (4\cos t, 4\sin t, -2\cos t)$

and then $\qquad \mathbf{F} \bullet \mathbf{r}'(t) = -8\sin t\cos t + 8\cos t\sin t - 8\cos^2 t$

$$\int_c \mathbf{F} \bullet \mathbf{ds} = \int_0^{2\pi} (-8\cos^2 t)dt = \left[-4\left(t + \tfrac{1}{2}\sin 2t\right)\right]_0^{2\pi} = -8\pi.$$

for 3)

$\mathbf{r}\,(t) = (t, t^3, 1)\,, t \in [-1, 0]$ $\mathbf{r'}\,(t) = (1, 3t^2, 1)$

$\|\mathbf{r'}(t)\| = \sqrt{2 + 9t^4}$

$$\int_c x^2 z \; ds = \int_{-1}^{0} (t^3\sqrt{1 + 9t^4}dt = (\text{ subst. } u = 2 + 9t^4, du = 36t^3 dt\ )$$

$$= \frac{1}{36}\int_{11}^{2} \sqrt{u}du = \frac{1}{18}\left[\frac{u^{\frac{3}{2}}}{3}\right]_{11}^{2} = \frac{1}{54}\left[2\sqrt{2} - 11\sqrt{11}\right]$$