The University of Calgary
Department of Mathematics and Statistics
MATH 353-02
Quiz #4T(3pm) Winter 2008

Name:____________________________**I.D.#:**__________________

1. Find the value of k so that the vector field
   $\mathbf{F}(x,y) = (4xy - \frac{1}{\sqrt{x}}, kx^2 + \ln y)$ is conservative for $x > 0, y > 0$
   then find a potential. [3]

2. Find $\int_c \mathbf{F} \bullet \mathbf{ds}$ where $\mathbf{F}(x,y,z) = (y^2 - z, x - y + 3, \frac{1}{z})$ and c is the part of the curve
   $\{\, z - y^2 = 1\} \cap \{x - y + 2z = 0\}$ from $A\,(-2,0,1)$ to $B\,(-5,-1,2)$. [4]

3. Evaluate $\int_c \frac{z}{x^2} ds$ [3]
   where c is given by $\mathbf{r}(t) = (e^t, (t-1)e^t, e^t)\,, t \in [0,1]\,.$

SOLUTION

For 1)

$F_1 = 4xy - \frac{1}{\sqrt{x}} \qquad F_2 = kx^2 + \ln y$

$(F_1)_y = 4x = (F_2)_x = 2kx \longrightarrow\longrightarrow k = 2$

looking for f such that $\quad f_x = 4xy - \frac{1}{\sqrt{x}} \qquad f_y = 2x^2 + \ln y$

integrate the first condition

$f = \int(4xy - \frac{1}{\sqrt{x}})dx + c(y) = 2x^2y - 2\sqrt{x} + c(y)$

differentiate w.r. to y and compare with the second cond.

$f_y = 2x^2 + c'(y) = 2x^2 + \ln y \qquad$ so $c'(y) = \ln y$

so $\;c = y\ln y - y$ and together $f\,(x,y) = 2x^2y - 2\sqrt{x} + y\ln y - y + const$

for 2)

we need to find a parametrization of c :choose $y = t$ then $z = 1 + t^2$, $x = t - 2 - 2t^2$

then for $A \qquad t = 0$, for $B \qquad t = -1$

and $\qquad \mathbf{r}(t) = (t - 2 - 2t^2, t, 1 + t^2) \quad \mathbf{r}'(t) = (1 - 4t, 1, 2t)$

we need the field $\mathbf{F}(x,y,z)$ on $c \qquad \mathbf{F} \circ \mathbf{r} = \left(-1, 1 - 2t^2, \frac{1}{1+t^2}\right)$

and then $\qquad \mathbf{F} \bullet \mathbf{r}'(t) = 4t - 2t^2 + \frac{2t}{t^2+1}$

$$\int_c \mathbf{F} \bullet \mathbf{ds} = \int_0^{-1} \left(4t - 2t^2 + \tfrac{2t}{t^2+1}\right) dt = \left[2t^2 - \frac{2}{3}t^3 + \ln(1+t^2)\right]_0^{-1} = 2 + \tfrac{2}{3} + \ln 2 = \frac{8}{3} + \ln 2.$$

for 3)

given: $\mathbf{r}(t) = (e^t, (t-1)e^t, e^t)$

$\mathbf{r}'(t) = (e^t, te^t, e^t) = e^t\,(1, t, 1) \qquad \|\mathbf{r}'(t)\| = e^t\sqrt{2+t^2}$

the given function f evaluated on $c \qquad f(\mathbf{r}(\mathbf{t}) = \dfrac{e^t}{e^{2t}} = e^{-t}$

then

$$\int_c f\ ds = \int_0^1 f(\mathbf{r}(\mathbf{t})\,\|\mathbf{r}'(t)\|\,dt = \int_0^1 \sqrt{2+t^2}\ dt = (\text{Table } a = \sqrt{2}) =$$

$$= \left[\tfrac{t}{2}\sqrt{2+t^2} + \ln(t + \sqrt{2+t^2}\right]_0^1 = \frac{\sqrt{3}}{2} + \ln\frac{1+\sqrt{3}}{\sqrt{2}}.$$