MATH 353 Handout #1 - Winter 2010

1. For each $S \subset R^2$, find ∂S. Is S open, closed, bounded? Sketch S as well.
   (a) $S = \{(x, y) : \frac{|x|}{|y|} \leq 1\}$,
   (b) $S = \{(x, y) : y - 2x = 1, 1 \leq y \leq 3\}$,
   (c) $S = \{(x, y) : y - 2x \leq 1, 1 \leq y \leq 3\}$,
   (d) $S = \{(x, y) : \ln(xy) \leq 0\}$,
   (e) $S = \{(x, y) : 0 < x^2 + y^2 < 4\}$,
   (f) $S = \{(x, y) : 0 < x^2 + y^2 \leq 4\}$.

2. Classify all critical points of $f(x, y) = 2xy^2 - x^2y + 4xy$. Find the absolute maximum, absolute minimum if it exists. Also, find the absolute maximum and absolute minimum of f on the three edges of the triangle with vertices $A = (0, 0)$, $B = (1, 0)$, $C = (0, 1)$.

3. Classify all critical points of $f(x, y) = 3y^3 - x^2y + x^2$.

4. **(Problem # 3 of ch 13.1 in the textbook)**. Find and classify all critical points of $f(x, y) = x^3 + y^3 - 3xy$.

5. **(Problem # 15 of ch 13.1 in the textbook)**. Find and classify all critical points of $f(x, y) = \left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{y}\right)\left(\frac{1}{x} + \frac{1}{y}\right)$.

6. **(Problem # 17 of ch 13.1 in the textbook)**. Find and classify all critical points of $f(x, y, z) = xy + x^2z - x^2 - y - z^2$.

7. **(Problem # 23 of ch 13.1 in the textbook)**. Postal regulations require that the sum of the height and girth(horizontal perimeter) of a package should not exceed L units. Find the largest volume of a rectangular box that can satisfy this requirement.