The University of Calgary
Department of Mathematics and Statistics
MATH 353 Handout #4 Answers

1. (a) $\beta = 2$
   (b) $\phi = x^3yz + y^2z + z$
2. (a) $\beta = 1/2$
   (b) $\phi = (1/2)xy^2 + (1/2)x^2 - 2\sqrt{y}$
3. (a) $\nabla\times\mathbf{F} = \langle 0.0.0\rangle$, $\nabla\bullet\mathbf{F} = 0$
   (b) (a) $\nabla\times\mathbf{F} = 2\langle yz, -xz, -xy\rangle$, $\nabla\bullet\mathbf{F} = y^2 - z^2 + x^2$
4. $\nabla\times\mathbf{F} = \langle 0.0.0\rangle$, $\nabla\bullet\mathbf{F} = 1/r = 1/\sqrt{x^2+y^2}$
5. Identities are all done pretty much the same way, just calculate carefully the LHS (left hand side) and the RHS and check both are equal. You can save a little work by just calculating the first coordinates, if they agree then the entire expressions will agree by symmetry. We will do 5(b) fully as an example.

   As usual write $\mathbf{F} = \langle P.Q.R\rangle$. Then $\nabla\times$ $\mathbf{F}$= $\langle R_2 - Q_3, P_3 - R_1, Q_1 - P_2\rangle$.

   This gives
   LHS= $\nabla\times(\nabla\times\mathbf{F}) = \langle Q_{12} - P_{22} - P_{33} + R_{13},\ \_\_\_,\ \_\_\_\rangle$.

   On the other hand
   RHS = $\nabla(P_1 + Q_2 + R_3) - \langle \nabla^2 P, \nabla^2 Q, \nabla^2 R\rangle$
   $= \langle P_{11} + Q_{12} + R_{13},\ \_\_\_,\ \_\_\_\rangle - \langle P_{11} + P_{22} + P_{33},\ \_\_\_,\ \_\_\_\rangle$
   $= \langle Q_{12} + R_{13} - P_{22} - P_{33},\ \_\_\_,\ \_\_\_\rangle$ = LHS
6. Another identity, same technique
7. By solving the two linear equations - Math 211 methods - one gets the solution with a single parameter : $x = 2t - 1, y = 2 - t, z = t$. Also, on the xy-plane $z = 0$, so $t = 0$, and at D it is clear $t = 2$. From the parametrization we have $\mathbf{v} = d\mathbf{r}/dt = \langle 2, -1, 1\rangle$. So $ds = \sqrt{6}dt$. The integral becomes

$$\int_0^2 t^2\sqrt{6}dt = \frac{8\sqrt{6}}{3}$$

8. From the given equations a good parametrization is given by choosing $y = t$, then it follows that $x = t^2$, $z = t + 1$. One finds that at A, $t = -1$, and at B, $t = 0$. By taking $\mathbf{v}$, one also finds that $ds = \sqrt{4t^2 + 2}dt$. So the integral becomes

$$\int_{-1}^{0} (t+1)\sqrt{4t^2+2}dt.$$

   This is a time consuming integral to work out, one will need a substitution like $t = (1/\sqrt{2})\tan\theta$. Or it's good practice to try it on MAPLE. The answer is

$$-\frac{1}{2}\ln(\sqrt{3} - \sqrt{2}) + \frac{1}{6}\sqrt{2}.$$