The University of Calgary
Department of Mathematics and Statistics
MATH 353 Handout #4

1. Given $\mathbf{F}(x, y, z) = \langle 3x^2yz, \beta yz + x^3z, x^3y + 1 + y^2\rangle$.

   (a) Find the value of β so that the field $\mathbf{F}$ is conservative.

   (b) Then, find a scalar potential of $\mathbf{F}$.

2. For $\mathbf{F}(x, y) = \langle \beta y^2 + x, xy - \frac{1}{\sqrt{y}}\rangle$, find the value for β so that the field is conservative, then find a potential.

3. Calculate **div F** and **curl F** for the following vector fields:

   (a) $\mathbf{F} = \text{y } \mathbf{i} + \text{x}\mathbf{j}$, (b) $\mathbf{F} = xy^2\mathbf{i} - yz^2\mathbf{j} + zx^2\mathbf{k}$

4. Calculate **div F** and **curl F** for the following vector fields in polar coordinates:

   $$\mathbf{F} = \hat{\mathbf{r}} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$$

5. Let ϕ and ψ be scalar fields and $\mathbf{F}$ and $\mathbf{G}$ be vector fields. Assume all are sufficiently smooth, prove the following identities:

   (a) $\nabla \bullet (\mathbf{F} \times \mathbf{G}) = (\nabla \times \mathbf{F}) \bullet \mathbf{G} - \mathbf{F} \bullet (\nabla \times \mathbf{G})$

   (b) $\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \bullet \mathbf{F}) - \nabla^2\mathbf{F}$

6. If ϕ and ψ are smooth scalar fields, show that

   $$\nabla \times (\phi\nabla\psi) = -\nabla \times (\psi\nabla\phi) = \nabla\phi \times \nabla\psi.$$

7. Evaluate $\int_c f\ ds$ where $f(x, y, z) = z^2$ and c is the part of the line of intersection of two planes $x + y - z = 1$ and $2x + y - 3z = 0$ between the xy-plane and the point $D = (3, 0, 2)$.

8. Evaluate $\int_c z\ ds$ where c is the intersection of the plane $z - y = 1$ and the cylindrical surface $0 = x - y^2$ between $A = (1, -1, 0)$ and $B = (0, 0, 1)$.