MATH 353 - Winter 2010
Handout #5

1. Find $\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r}$ where $\mathbf{F}(x, y, z) = \langle y, x + z, y - 2z \rangle$ and $\mathcal{C}$ is the intersection of the plane $z = 2x$ and the paraboloid $z = x^2 + y^2$. [Hint : test the vector field $\mathbf{F}$ to see if it is conservative.]

2. Find $\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r}$ where $\mathbf{F}(x, y, z) = \langle z, e^{(y/x)}, 2x \rangle$ and $\mathcal{C}$ is given by $\mathbf{r}(t) = \langle t, t^2, e^t \rangle$, $0 \le t \le 1$.

3. Find $\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle ye^{xy}, xe^{xy} + x \rangle$ and $\mathcal{C}$ is the circle $x^2 + y^2 = 1$, oriented counterclockwise. [Hint : you can save a lot of work by noticing that part of the vector field $\mathbf{F}$ is conservative.]

4. Find the surface area of $\mathcal{S}$

   (a) which is the part of the cylinder $x^2 + y^2 = 4$ in the first octant below the plane $2x + y + z = 5$;

   (b) which is the part of the plane $2x + y + z = 5$ inside the cylinder $x^2 + y^2 = 4$.

5. Evaluate $\int\int_{\mathcal{S}} zx \, dS$ where $\mathcal{S}$ is the part of $z = \frac{x^2}{2}$ which lies inside $x^2 + y^2 = 1$, $x > 0$, $y < 0$.

6. Evaluate $\int\int_{\mathcal{S}} x^2 dS$ where $\mathcal{S}$ is the part of the plane $x + y + z = 2$ inside the cylinder $x^2 + 2y^2 = 1$.

7. Find the flux of $\mathbf{F} = \mathbf{i} + \mathbf{j} + z(x^2 + y^2)^2\mathbf{k}$ out of the closed surface (including the top and bottom of the cylinder) $\mathcal{S} = \{(x, y, z) : x^2 + y^2 = 4, \ 0 \le z \le 3\}$.

8. Find the flux $\int\int_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$ where $\mathcal{S}$ is the part of the cylinder $y^2 + z^2 = 4$ which lies inside the cylinder $x^2 + y^2 = 4$, $y, z \ge 0$, oriented upward, and the field is $\mathbf{F}(x, y, z) = \langle x^2yz, y, xz \rangle$.