MATH 353 - Winter 2010
Handout #6

1. Use Green's theorem to evaluate $\int_{\mathcal{C}} x^2y^2\ dx + 4xy^3\ dy$ where $\mathcal{C}$ is the triangle with vertices $(0,0)$, $(1,3)$ and $(0,3)$, oriented positively.

2. Use Green's thereom to evaluate $\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r}$ where

$$\mathbf{F}(x,y) = \langle \sqrt{x} + y^3, x^2 + \sqrt{y} \rangle$$

   and $\mathcal{C}$ consists of the arc of the curve $y = \sin x$ from $(0,0)$ to $(\pi, 0)$.

3. Use the divergence theorem to calculate the flux of

$$\mathbf{F}(x,y,z) = \langle 4x^3z, 4y^3z, 3z^4 \rangle$$

   out of the sphere $\mathcal{S}$ with radius R centred at the origin.

4. Use the (two-dimensional) divergence theorem to evaluate $\int_{\mathcal{C}} \mathbf{F} \bullet \hat{N} ds$ where $\mathbf{F}(x,y) = -y\mathbf{i} + x\mathbf{j}$ and $\mathcal{C}$ is given as $x^2 + y^2 = 1$, oriented counterclockwise.

5. Use Stokes's theorem to evaluate $\int\int_{\mathcal{S}} \nabla \times \mathbf{F} \bullet d\mathbf{S}$ where $\mathbf{F}(x,y,z) = \langle yz, xz, xy \rangle$ and $\mathcal{S}$ is the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$, oriented upward.

6. Use Stokes's theorem to evaluate $\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r}$ where $\mathbf{F} = \langle e^{-x}, e^x, e^z \rangle$ and $\mathcal{C}$ is the boundary of the part of the plane $2x + y + 2z = 2$ in the first octant, oriented by the direction from $(1,0,0)$ to $(0,2,0)$.