MATHEMATICS 353

Winter 2010
REVIEW FOR FINAL EXAM

1. A few Basics - to be done at the review session only.
2. Max and min for $f(x,y) = 2x^2 - 3y^2 - 2x$ on the closed unit disc $\mathcal{D}$.
3. Let $\mathcal{D}$ be the entire second quadrant. Determine
$$\int\int_{\mathcal{D}} xe^{x-y}\ dA.$$
4. Evaluate the integral
$$\int\int_{\mathcal{D}} e^{3(x^2+y^2)}\ dxdy, \text{ where } \mathcal{D} = \{(x,y)\ :\ 0 \le y,\ 1 \le x^2+y^2 \le 4\}.$$
5. Express the following iterated integral in the order $dxdzdy$:
$$\int_0^1 dx \int_0^{1-x} dy \int_y^1 f(x,y,z)\ dz.$$
6. If ϕ is a scalar field and $\mathbf{F}$ is a vector field, prove that
$$\nabla \times (\phi\mathbf{F}) = (\nabla\phi) \times \mathbf{F} + \phi(\nabla \times \mathbf{F}).$$
7. Evaluate $\int_{\mathcal{C}} f\ ds$, where $f(x,y,z) = y + z^2 - 1$ and $\mathcal{C}$ is the portion of the curve $\mathbf{r}(t) = \langle (4/3)t^{3/2},\ t^2+1,\ t\rangle$ from $A = (0,1,0)$ to $B = (4/3, 2, 1)$.
8. Given $\mathbf{F}(x,y,z) = \langle \alpha x + yz,\ xz,\ xy\rangle$, find α so that $\mathbf{F}$ is conservative, and then find a potential function ϕ.
9. Find the surface area of $\mathcal{S}$, the part of the cylinder $x^2+y^2 = 4$ that is in the first octant and below the plane $x + y + 2z = 6$.
10. Evaluate $\int_{\mathcal{C}} 3x^2y^2\ dx + 4x^3y\ dy$, where $\mathcal{C}$ is the boundary of the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$, oriented positively.
11. Find the flux of the vector field $\mathbf{F}(x,y,z) = y\mathbf{i} - x\mathbf{j} + 4\mathbf{k}$ upward through the surface $\mathcal{S}$, where $\mathcal{S}$ is the part of the surface $z = 1 - x^2 - y^2$ that lies inside the cylinder $x^2 + y^2 = 1$.
12. Evaluate $\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r}$, where $\mathbf{F} = \langle ye^x,\ x^2 + e^x,\ z^2e^z\rangle$ and $\mathcal{C}$ is the curve $\mathbf{r}(t) = \langle 1 + \cos t,\ 1 + \sin t,\ 1 - \cos t - \sin t\rangle$, $0 \le t \le 2\pi$. [Hint : Use Stokes's theorem.]