1. In the following questions $\phi, \psi$ are scalar fields and $\mathbf{F}, \mathbf{G}$ are vector fields in $\mathbb{R}^3$. All functions are assumed to be smooth. For each question, write either "vector field," "scalar field," or "meaningless" in the space provided.

   (a) $\nabla \bullet (\nabla \phi)$ ________________

   (b) $(\nabla \bullet \mathbf{F}) \times (\nabla \bullet \mathbf{G})$ ________________

   (c) $(\nabla \times \mathbf{F}) \times \mathbf{G}$ ________________

   (d) $\nabla \bullet (\nabla \times \phi)$ ________________

   (e) $(\nabla \bullet \mathbf{F})\, \mathbf{G}$ ________________

2. For each of the following answer "True" or "False". Do not write "T" or "F".

   (a) $\{(x, y, z) : x + y + z < 10 \text{ and } x^2 + y^2 + z^2 \leq 3\}$ is closed.

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   (b) The matrix

   $$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & \sqrt{2} \\ 0 & \sqrt{2} & 6 \end{bmatrix}$$

   is positive definite. ________________

   (c) At any point of a smooth surface $\mathcal{S}$ in $\mathbb{R}^3$ there is a unique unit normal vector $\mathbf{N}$.

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   (d) If $\mathbf{F}$ is a conservative vector field, defined over a simply connected domain $\mathcal{D}$, then $\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r} = 0$ for any oriented path $\mathcal{C}$ in the domain $\mathcal{D}$ .

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   (e) The following equality is correct. ________________

   $$\int_0^{\pi} \int_0^{\pi/4} \int_2^3 e^{\theta+\phi} \ln(\rho) \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$
   $$= \left( \int_0^{\pi} e^{\theta} \, d\theta \right) \cdot \left( \int_0^{\pi/4} e^{\phi} \sin\phi \, d\phi \right) \cdot \left( \int_2^3 \ln(\rho) \cdot \rho^2 \, d\rho \right)$$

For Questions 3-10 circle the correct answer. Your must show your work.

3. The set of values for $c$ such that the matrix

$$\begin{bmatrix} 1 & 3 & 0 \\ 3 & 10 & c \\ 0 & c & 4 \end{bmatrix}$$

is positive definite is given by

(a) $-\sqrt{40} \leq c \leq \sqrt{40}$ (b) $-2 \leq c \leq 2$

(c) $-\sqrt{40} < c < \sqrt{40}$ (d) $-2 < c < 2$

(e) $0 \leq c \leq 40$.

4. Let $\mathcal{D}$ be the triangular domain given by $0 \leq y \leq 3,\ \ (y/3) - 1 \leq x \leq 1 - (y/3)$. Then

$$\int\int_{\mathcal{D}} \left(e - x^5 e^{\sqrt{1+y^2}}\right) \, dA =$$

(a) $3e$ (b) $0$

(c) $6e$ (d) $e - e^{\sqrt{244}}$

(e) Undefined.

5. Let $\mathcal{R}$ be the solid ball given by $x^2 + (y-2)^2 + (z+4)^2 \leq 1$, let $\mathcal{S} = \partial\mathcal{R}$, oriented by the outward normal, and let $\mathbf{F}(x,y,z) = \langle 2x\,,\, y+\cos(z)\,,\, 3z\rangle$, then the flux $\displaystyle\int\int_{\mathcal{S}} \mathbf{F} \bullet d\mathbf{S}$ equals

(a) $2\pi$ (b) $4\pi$

(c) $6\pi$ (d) $8\pi$

(e) 0.

6. Let $\mathcal{R}$ be the region given by $1 \leq x^2 + y^2 + z^2 \leq 4$ and $z \geq 0$, then $\displaystyle\int\int\int_{\mathcal{R}} 3\cos\left((x^2+y^2+z^2)^{\frac{3}{2}}\right)\, dV$ equals

(a) $2\pi[\sin(8) - 1]$ (b) $\pi[\sin(8) - 1]$

(c) $2\pi[\sin(8) - \sin(1)]$ (d) $\pi[\sin(8) - \sin(1)]$

(e) 0

7. Let $\mathcal{D}$ be the entire first quadrant $x, y \geq 0$ in the $x$-$y$ plane. The value of $\displaystyle\int\int_{\mathcal{D}} xe^{-x^2-y}\,dA$ equals

(a) $-2$ (b) $1/2$

(c) $2$ (d) $-1/2$

(e) Undefined.

8. Given that the vector field $\mathbf{F}(x, y, z) = \langle 2x\cos(y)\ ,\ -x^2\sin(y) + e^z\ ,\ ye^z\rangle$ is conservative, find the line integral $\displaystyle\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r}$, from the point $A = (0, 0, 0)$ to the point $B = (1, \pi, \ln(2))$ along the path $\mathbf{r}(t) = \langle t^{5/3}\ ,\ 4\arctan(t)\ ,\ \ln\sqrt{3t^2 + 1}\rangle$. (Hint : Find a potential function $\phi$ for $\mathbf{F}$)

(a) $-1 + 2\pi$ (b) $1 + \pi + \ln(2)$

(c) $1 + \pi$ (d) $0$

(e) $4\arctan(\pi)$.

9. Consider the function $f(x, y) = \dfrac{1}{x^2 + y^2}$ defined on the domain $\mathcal{D} = \{(x, y) \ : \ x^2 - 2x + y^2 \leq 0\}$ (Hint: It is a closed disk of radius 1). Which of the following statements is true?

(a) $f$ has maximum 1 and no minimum

(b) $f$ has maximum 2 and no minimum

(c) $f$ has no maximum and minimum 1

(d) $f$ has maximum 4 and minimum 1/4

(e) $f$ has no maximum and minimum 1/4

10. Let $\mathcal{C}$ be the closed curve in $\mathbb{R}^2$ joining, by straight line segments, the points $(-1, 1)$, $(3, 0)$, $(1, 4)$, $(-2, 2)$ and back to $(-1, 1)$ (in the given order). If $\mathbf{F}(x, y) = \langle e^x y^2/2 + \arctan(x), e^x y + 2y \rangle$, then $\displaystyle\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r}$ equals

(a) 4 (b) 3

(c) $e^4$ (d) $e^3$

(e) 0

11. Determine the volume of the region bounded above by the paraboloid $z = 10 - x^2 - y^2$ and below by the cone $z^2 = 9(x^2 + y^2)$, with $z \geq 0$.

12. Find and classify the critical point(s) of the function

$$f(x, y) = \frac{x^3}{2} + \frac{y^3}{2} - 3xy + 2.$$

13. A box without top is made of material for the bottom costing $\$5/m^2$, the front and back $\$1/m^2$, and the sides $\$2/m^2$. The total cost is fixed at \$ 3,000. Find the dimensions that will maximize the volume.

14. Set up, but **do not** evaluate, a double integral (including the limits of integration) computing the surface area of $\mathcal{S}$ which is the part of the cylinder $x^2+4y^2 = 4$ in the **first octant** below the plane $x+y+2z = 3$.

15. Evaluate $\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r}$ where $\mathbf{F} = \langle ye^x, x + e^x, z^2e^z \rangle$ and $\mathcal{C}$ is the curve which is the intersection of the plane $z = 3 - x - y$ and the cylinder $x^2 + y^2 = 1$, oriented from the point $(1, 0, 2)$ on the curve to the point $(0, 1, 2)$ on the curve. [Hint: Use Stokes's Theorem]