

Mathematics 411

Linear Spaces with Applications

(see Course Descriptions for the applicable academic year: <http://www.ucalgary.ca/pubs/calendar/>)

Syllabus

<u>Topics</u>	<u>Number of Hours</u>
Fields, algebras, vector spaces	3
Elementary canonical forms: invariant subspaces, simultaneous diagonalization, direct-sum decompositions, invariant direct sums.	8
Rational and Jordan forms.	6
Inner product spaces, adjoint, Hermitian, unitary and normal operators.	8
Operators on inner product spaces, form, the Spectral theorem.	6
Bilinear and quadratic forms.	5
TOTAL HOURS	36

MATH 411- Linear Spaces with Applications

Course outcomes

At the end of the course, students should be able to:

1. prove statements describing basic properties of a matrix connecting matrix properties with its eigenvalues, illustrate solution of systems of linear equations over various fields
2. describe linear operators in various bases, as well as transition matrices, kernels and images of linear operators, to construct a dual basis for a basis of a finite-dimensional space;
3. establish similarity of matrices, to identify whether a given square matrix is diagonal-izable or not on a given field and to find a basis, if possible, in which a given operator has a diagonal form, to compute the minimal polynomial and a Jordan form of a linear operator on a finite dimensional vector space, and to use shortcuts to compute expressions with square block matrices more efficiently;
4. compute an orthogonal basis of a subspace of an inner product space, to find coefficients of orthogonal expansions, apply Schwarz and triangle inequalities to verify algebraic and functional inequalities;
5. compute adjoint operators, to identify unitary, Hermitian, positive definite and normal operators;
6. identify whether two symmetric matrices are congruent, to compute canonical forms and apply positive and negative definiteness of quadratic forms to the analysis of local extrema for functions of several variables;
7. compute LU,LDU decomposition of square matrices, as well as QR factorization when the number of columns does not exceed the number of rows, implement Singular Value Decomposition and find a Moore-Penrose pseudoinverse of an arbitrary matrix, as well as apply these methods in numerical analysis, for example, to solve least squares problems.

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