UNIVERSITY OF CALGARY

FACULTY OF SCIENCE
Department of Mathematics and Statistics

Mathematics 545 Analysis III

(see Section 3.5C of Faculty of Science www.ucalgary.ca/pubs/calendar/current/sc-3-5.html and Course Descriptions: http://www.ucalgary.ca/pubs/calendar/current/course-main.html)

Syllabus

Topics	**Number of hours**
Sequences and series of functions; pointwise and uniform convergence; Weierstrass M-test	3
Differentiation and integration of series; power series	3
Step functions and their integrals; integration of limits of increasing sequences of step functions	3
The Lebesgue integral and its basic properties; sets of measure zero	3
The monotone and dominated convergence theorems; Fatou's lemma	3
Functions defined by integrals and differentiation under the integral sign; Fubini's theorem	3
Square-integrable functions; completeness of L^2; Hilbert space axioms	3
The Hilbert space l^2; Fourier series as an isometry of L^2 with l^2; self-duality of Hilbert spaces	3
The Fourier series of a function; Parseval's formula; the Riesz-Fischer theorem; The L^2-density of trigonometric polynomials, Riemann-Lebesgue lemma	3
Pointwise convergence of Fourier series	3
The Fourier transform and its properties; the Fourier integral theorem	3
Convolution and the Fourier transform; the Laplace transform; applications to differential equations	3
Further topics, e.g., the Dirac delta function and its Fourier transform (time permitting)	
TOTAL HOURS	**36**

Course Outcomes

Overview

This course aims to introduce the students to the basic concepts of Function Theory, Fourier Analysis, the Theory of Lebesgue Integration and L^p spaces. In particular, many notions of the Analysis II course will be generalized to more general settings of Hilbert spaces, function algebras and measure theory. At the conclusion of this course, students are expected to

1. Apply different notions and techniques studied in the course to solution of basic problems of Functional Analysis and Function Theory.

2. Clearly understand main ideas of proofs of results presented in the course and be able to apply similar methods and ideas to problems offered on written tests.

3. Learn basic concepts and techniques of the Measure theory and the theory of the Lebesgue integral.

4. Be able to solve basic problems from the Textbook on Fourier Transforms, Fourier series, the Plancherel and the Parseval theorems.

5. Be able to present in class some topics assigned during the term, answer questions and solve problems related to them.

Students of MATH 603 are expected to have sufficient knowledge and practice to successfully pass the Ph.D. Preliminary Exam in Real Analysis.

Subject Specific Knowledge
By the end of this course, students are expected to

6. Be able to solve different problems on pointwise and uniform convergence of sequences of functions and function series and to apply the Stone-Weierstrass theorem to approximate ‘complicated’ continuous functions by simpler ones.

7. Be able to solve basic problems of the Fourier Theory on the real line.

8. Understand the main concepts and ideas of the proofs of the theory of the Lebesgue measure on the real line and the diff between measurable and non-measurable functions. Be able to solve basic problems involving Cantor sets and Cantor functions and to apply diff t techniques of the Lebesgue integral theory (such as the Lebesgue dominated convergence theorem and the Fatou lemma).

9. Be able to state the axioms of L^p spaces on the real line and prove their completeness. Also, be able to state the axioms of the Hilbert space theory, prove the triangle and Cauchy-Schwarz inequalities, apply the Gram-Schmidt orthogonalization method to construct

orthogonal polynomials (Hermite, Laguerre, Legendre polynomials).

10. Be able to solve basic problems on the Fourier transform and the Plancherel Theorem.

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