PMAT 315 Winter 2007

ASSIGNMENT 2

Due 4:00 PM Friday, March 2, 2007. Put your assignment in the appropriate **wooden slot** (corresponding to your lecture section and last name) inside room MS 315. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words.

Marked assignments will be handed back during your scheduled tutorial, or in class.

1. (a) Page 68 #20.

(b) Of course for any group G, G and $\{e\}$ are subgroups of G. What are $C(G)$ and $C(\{e\})$?

(c) Suppose that H_1 and H_2 are subgroups of a group G and that $H_1 \subseteq H_2$. Prove that $C(H_1) \supseteq C(H_2)$.

(d) Suppose that $G = S_3$, the symmetric group of degree 3, and $H = A_3$, the alternating group of degree 3. Find $C(A_3)$.

2. (a) Page 83 #19. (See Example 11 page 46.) Be sure to list all the elements of each cyclic subgroup and how they are generated.

(b) Find a subgroup of $U(30)$ of order 4 which is *not* cyclic but is a union of cyclic subgroups.

3. Page 85 #62. Also give (with proof of course) an isomorphism from H to $\mathbb{Z}$.

4. (a) Page 115 #50.

(b) Relabel the cards $\{A, 2, \ldots, Q, K\}$ as the integers $\{1, 2, \ldots, 13\}$ in that order. Use this relabelling to write your answer to part (a) as a permutation of $\{1, 2, \ldots, 13\}$ in array form. Then write this permutation as a product of disjoint cycles. Is this permutation odd or even?

5. (a) Suppose G is an Abelian group with no element of order 2. Prove that for all $g, h \in G$, if $g^2 = h^2$ then $g = h$.

(b) Page 134 #30.

(c) Give the automorphism of part (b) in the case $G = (\mathbb{Z}_5, +)$. [*Warning*: in part (b) the group notation is multiplicative.]