PMAT 315 Winter 2008

ASSIGNMENT 1

Due **4:00 PM Friday February 8**. You may hand your assignments to me (in class or in my office MS566), or to the marker Zsolt Langi in MS490. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words.

1. (a) Page 26 # 49.

(b) Suppose instead that S is a *subset* of $\mathbb{Z}$. Again for $a, b \in S$ define aRb if $ab \geq 0$. Find all subsets $S \subseteq \mathbb{Z}$ so that R is an equivalence relation on S. Also, for each $S \subseteq \mathbb{Z}$ so that R is an equivalence relation on S, describe all the equivalence classes of R.

2. (a) For any $a, b \in \mathbb{R} - \{0\}$, let $a \circ b = -2ab$. Prove that $(\mathbb{R} - \{0\}, \circ)$ is a group.

(b) Prove that we do not get a group if $\mathbb{R} - \{0\}$ in part (a) is replaced by $\mathbb{R}$ or by $(0, \infty)$ (the set of all positive real numbers).

(c) Find a nontrivial proper subgroup of the group $(\mathbb{R} - \{0\}, \circ)$.

3. A *mattress* is a solid rectangular box whose length, width and height are all different. Find all symmetries of a mattress (using appropriate labels). Then give the Cayley table of the group of symmetries of a mattress, and give the orders of all the elements of this group.

4. Recall that for any element a of a group G, and for any positive integer n, a^{-n} is defined to be $(a^{-1})^n$. Prove **by induction on** n that $a^{-n} = (a^n)^{-1}$ for all positive integers n.

5. (a) Page 91 #8.

(b) Prove that for any elements a, b of a group G, if $ab = c^2$ for some $c \in G$, then $ba = d^2$ for some $d \in G$.

(c) Find an example of a group G and elements $a, b \in G$ so that $ab \neq c^2$ for any $c \in G$.