PMAT 315 Winter 2008

ASSIGNMENT 2

Due **4:00 PM Friday March 7**. You may hand your assignments to me (in class or in my office MS566), or to the marker Zsolt Langi in MS490. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words.

1. (a) Let $n \geq 3$ be an integer, and consider the transposition

$$(12) = \begin{bmatrix} 1 & 2 & 3 & \dots & n \\ 2 & 1 & 3 & \dots & n \end{bmatrix}$$

in the symmetric group S_n. Suppose that some permutation $\sigma \in S_n$ satisfies $\sigma(12) = (12)\sigma$. Find all possible values of $\sigma(1)$ and $\sigma(2)$.

(b) Use part (a) to describe all elements of the centralizer $C((12))$ of the element (12) of S_n, $n \geq 3$. [See the definition on page 66.]

(c) Page 114 #46.

2. Let the *mattress group* M be the group of symmetries of a mattress, as given in problem 3 of Assignment 1. [*Note*: M has only **four** elements. See the solution posted on the course website.]

(a) Is $M \approx \mathbb{Z}_4$? Explain.

(b) Use the construction in the proof of Cayley's Theorem to find four permutations of a four-element set which form a group isomorphic to M.

3. Let G be a group, and define $\phi : G \to G$ by $\phi(g) = g^2$ for all $g \in G$.

(a) Is ϕ an automorphism of G for every group G? Explain.

(b) Is ϕ an automorphism of G for every Abelian group G? Explain.

(c) Is ϕ an automorphism of G for $G = (\mathbb{R}^+, \cdot)$? Explain.

4. Let $G = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ and $H = \left\{ \begin{bmatrix} a & 2b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{Z} \right\}$.

(a) Prove that G is a subgroup of $(\mathbb{R}, +)$.

(b) Prove that $(H, +)$ is a group, where the operation is the usual addition of matrices.

(c) Prove that $G \approx H$.

(d) Find two elements in $\mathrm{Aut}(G)$ which are not the identity automorphism.

(e) Is G cyclic? Explain.