PMAT 315 Winter 2008

ASSIGNMENT 3

Due **4:00 PM Friday March 28**. You may hand your assignments to me (in class or in my office MS566), or to the marker Zsolt Langi in MS490. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words.

1. (a) Let $G \approx G'$ be isomorphic groups and let H be a subgroup of G. Prove that G' has a subgroup H' which is isomorphic to H.

(b) Page 149 #22.

2. (a) Let G be a finite group with $|G|$ odd. Prove that every element of G has odd order.

(b) Page 149 #30.

(c) Give an example of as noncyclic group of odd order.

3. (a) Page 165 #4.

(b) Page 166 #20.

(c) Actually, it turns out that $D_6 \approx S_3 \oplus \mathbb{Z}_2$. Prove this using the "internal direct product" theorem we did in class (page 189, Theorem 9.6 with $n = 2$). That is, find normal subgroups H and K of D_6 so that $H \approx S_3$, $K \approx \mathbb{Z}_2$, $H \cap K = \{e\}$, and $HK = D_6$. You may use Example 11 on page 66 and Example 2 on page 178.

4. (a) Page 193 #40.

(b) Give an example of an abelian group G (with $|G| > 1$) and a nontrivial proper subgroup H of G so that every element of H and every element of G/H is a square.

5. (a) Page 194 #49. (See pages 32–33 for the notation.)

(b) Page 194 #53.