PMAT 315 Winter 2008

ASSIGNMENT 4

Due **4:00 PM Friday April 18**. You may hand your assignments to me (in class or in my office MS566), or to the marker Zsolt Langi in MS490. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words.

1. (a) Page 213 #52.

(b) In part (a), suppose that $G = \overline{G} = S_3$, and define $\alpha : S_3 \to S_3$ and $\beta : S_3 \to S_3$ by: α is the identity function, and $\beta(\sigma) = (12)\sigma(12)^{-1}$ for all $\sigma \in S_3$. (So β is the inner automorphism of S_3 induced by the transposition (12); see page 129.) We know that α and β are automorphisms of S_3, so they are group homomorphisms. Find H in part (a) in this case. [*Hint*: you could use the result of Assignment 2, # 1(b).]

(c) In part (a), must H be a normal subgroup of G?

2. Let $G = \mathbb{Q} \cap [0,1)$ be the set of all rational numbers greater than or equal to 0 and less than 1, and define the operation $+_1$ on G to be addition modulo 1: that is, for $q_1, q_2 \in G$, $q_1 +_1 q_2 = \begin{cases} q_1 + q_2 & \text{if } q_1 + q_2 < 1, \\ q_1 + q_2 - 1 & \text{if } q_1 + q_2 \geq 1. \end{cases}$

(a) Prove that $(G, +_1)$ is a group.

(b) For each rational number q, let r_q be the fractional part of q, that is, r_q is the unique rational number satisfying $0 \leq r_q < 1$ and $q - r_q$ is an integer. Let $\phi : \mathbb{Q} \to G$ be defined by: for all $q \in \mathbb{Q}$, $\phi(q) = r_q$. Prove that ϕ is an onto group homomorphism.

(c) Page 230 #11. Do it using part (b) and the First Isomorphism Theorem for groups (Theorem 10.3).

3. (a) Page 241 #18.

(b) Find S (in part (a)) when $R = M_2(\mathbb{Z})$ and $a = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. Is S an ideal of $M_2(\mathbb{Z})$?

(c) Suppose R is a commutative ring. Prove that S (in part (a)) is an ideal.

(d) Suppose R is an integral domain. Find all possibilities for S in part (a).

4. (a) Page 255 #16.

(b) Page 257 #52.

5. Page 269 #24.