PMAT 315

ASSIGNMENT 2 WINTER 2010

1. §1.4, #25. *Show that every even permutation is a product of 3-cycles.* 8 marks

2. §2.1, #18 *Show that the following are equivalent for a monoid M :* 8 marks

   (1) *If ab is a unit then both a and b are units.*

   (2) *If $ab = 1$ then $ba = 1$.*

3. *Let H and K denote subgroups of a group G.*

   (1) *Show that $H \cap K$ is a subgroup of G.* 2 marks

   (2) *Show that $H \cup K$ is a subgroup of G if and only if $H \subseteq K$ or $K \subseteq H$.* (§2.3, #17) 6 marks

4. (a) *Find the order of $\bar{2}$ in $\mathbb{Z}_{11}^{*}$.* 4 marks

   (b) *Find the order of $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 8 & 9 & 7 & 6 & 3 & 4 & 1 & 5 \end{pmatrix}$ in S_9.* 4 marks

5. *Let a be an element of order $|a| = n$ in a group G. Given any integer m, let $d = gcd(n, m)$. Show that $\langle a^m \rangle = \langle a^d \rangle$.* 8 marks

Total 40 marks