PMAT 315

ASSIGNMENT 3 WINTER 2010

1. §2.5, #28. *Show that* $\mathbb{R}^+ \times \mathbb{C}^0 \cong \mathbb{C}^*$ *where* $\mathbb{R}^+ = \{r \in \mathbb{R} \mid r > 0\}$ *and* $\mathbb{C}^0 = \{z \in \mathbb{C} \mid |z| = 1\}$.
   [Hint: If $z \in C^*$ then $z = re^{i\theta}$ where $r > 0$.] 8 marks

2. *Show that* $(\mathbb{R}, +)$ *is not isomorphic to* $\mathbb{R}^*$. [Hint: If $\sigma : \mathbb{R} \to \mathbb{R}^*$ is an isomorphism, there exists $r \in \mathbb{R}$ such that $\sigma(r) = -1$.] 8 marks

3. *Let* H *and* K *be finite subgroups of some group* G.
   (1) *If* $|H|$ *and* $|K|$ *are relatively prime, show that* $H \cap K = \{1\}$. 6 marks
   (2) *If* $H \cap K = \{1\}$, *does it follow that* $|H|$ *and* $|K|$ *are relatively prime? Justify your answer.* 2 marks

4. §2.4, #26(a). *Let* $|g| = m$ *and* $|h| = n$ *in a group* G *where* m *and* n *are relatively prime. If* $gh = hg$, *show that* $|gh| = mn$. *Is* $|gh| = lcm(m, n)$ *in general?* 8 marks

5. *If a group* G *has subgroups or orders* 45 *and* 75, *and if* $|G| \leq 400$, *determine* $|G|$. 8 marks

Total 40 marks