PMAT 315
ASSIGNMENT 4 WINTER 2010

1. *Let G be a group with $|G| = p^2$ where p is a prime. Show that G is abelian. You may use the known fact that $Z(G) \neq \{1\}$ for such a group.* 8 marks

2. *Write $G = D_6 = \{1, a, a^2, a^3, a^4, a^5, b, ba, ba^2, ba^3, ba^4, ba^5\}$ where, as usual $|a| = 6$, $|b| = 2$ and $aba = b$. Consider the subgroup $K = \langle a^2 \rangle = \{1, a^2, a^4\}$.*

   (1) *Show that $K \lhd G$.* 4 marks

   (2) *Show that $G' = K$.* 4 marks

3. §2.9, #7. *Show that $\mathbb{Q}/\mathbb{Z}$ is an infinite abelian group in which every element has finite order.* 8 marks

4. §2.10, #17. *Let G be an abelian group. Let $T(G)$ denote the set of elements of G of finite order, and call G* **torsion-free** *if $T(G) = \{1\}$, that is if 1 is the only element of finite order.*

   (1) *Show that $T(G)$ is a subgroup of G.* 4 marks

   (2) *Show that $G/T(G)$ is torsion-free for any abelian group G.* 4 marks

5. In each case use the isomorphism theorem.

   (1) Let $G = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \in \mathbb{R}, a \neq 0 \neq c \right\}$, *a group using matrix multiplication. If $K = \left\{ \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \mid b \in \mathbb{R} \right\}$, show that $K \lhd G$ and $G/K \cong \mathbb{R}^* \times \mathbb{R}^*$.* 4 marks

   (2) §2.10, # 21. *Show that $\mathbb{C}^*/\mathbb{C}^0 \cong \mathbb{R}^+$ where $\mathbb{R}^+ = \{r \in \mathbb{R} \mid r > 0\}$ under multiplication.* 4 marks

Total 40 marks