PMAT 315
ASSIGNMENT 6 WINTER 2010

1. (a) For which primes p is $x+2$ a factor of $f(x) = 5x^4 - 2x^3 + 3x^2 + 4x - 1$ in $\mathbb{Z}_p[x]$? 3 marks

   (b) Factor $f(x) = x^3 + 1$ into linear factors in $\mathbb{Z}_7[x]$. 2 marks

   (c) Find all roots in $\mathbb{Q}$ of $f(x) = 4x^4 + 4x^3 + 3x^2 - x - 1$, and factor $f(x)$ as far as possible in $\mathbb{Q}[x]$. 3 marks

2. §4.1, #26. Show that $\sqrt[n]{m}$ is not rational unless $m = k^n$ for some integer k. 8 marks

3. (a) Factor $f(x) = x^4 - x^2 + x - 1$ into irreducibles in $\mathbb{Z}_{13}[x]$. 2 marks

   (b). Factor $f(x) = x^5 + 6x^4 + 12x + 15$ into irreducibles in $\mathbb{Q}[x]$. 2 marks

   (c) Factor $f(x) = x^4 - x^3 + 2x^2 - 3x + 2$ into irreducibles in $\mathbb{Q}[x]$. 4 marks

4. §4.3, #10(a). If F is any field, show that $\frac{F[x]}{\langle x^2 \rangle} \cong \left\{ \left[\begin{smallmatrix} a & b \\ 0 & a \end{smallmatrix} \right] \middle| a, b \in F \right\}$. 8 marks

5. §4.3, #14(d). If $p(x) = x^3 - x^2 + 1$, let $E = \mathbb{Z}_3[x]/\langle p(x) \rangle$. Factor $p(x)$ into linear factors in $E[x]$. 8 marks