PMAT 315
ASSIGNMENT 1 WINTER 2011

1. §1.2, #14. *Show that* $gcd(m+n,m) = gcd(m,n)$. 8 marks

2. §1.2, #17. *If* $gcd(m,n) = 1$ *and* $gcd(k,n) = 1$, *show that* $gcd(mk,n) = 1$. 8 marks

3. §1.3, #22(c). *If* $d_1, d_2, \ldots, d_r$ *are all divisors of* n, *and if* $gcd(d_i, d_j) = 1$ *whenever* $i \neq j$, *show that the product* $d_1 d_2 \cdots d_r$ *divides* n. 8 marks

4. §1.3, #29(b). *Show that* $\bar{a}$ *is invertible in* $\mathbb{Z}_n$ *if and only if* $\bar{a}\bar{b} = \bar{0}$ *implies* $\bar{b} = \bar{0}$. 8 marks

5. §1.4, #26. *Let* γ *be any cycle of length* r.

   (1). *If* $\sigma \in S_n$, *show that* $\sigma\gamma\sigma^{-1}$ *is also a cycle of length* r. *More precisely, if* $\gamma = (k_1\ k_2\ \cdots\ k_r)$ *show that* $\sigma\gamma\sigma^{-1} = (\sigma k_1\ \sigma k_2\ \cdots\ \sigma k_r)$. 4 marks

   (2) *If* $\tau \in S_n$, *show that* τ *and* $\sigma\tau\sigma^{-1}$ *have the same "cycle structure"; that is, in the factorization into disjoint cycles they have the same number of cycles of each length.* [Hint: $\sigma\gamma\delta\sigma^{-1} = (\sigma\gamma\sigma^{-1})(\sigma\delta\sigma^{-1})$.] 4 marks

Total 40 marks