PMAT 315

ASSIGNMENT 2 WINTER 2011

1. §2.1, #18 *Show that the following are equivalent for a monoid* M : 8 marks

   (1) *If* ab *is a unit then both* a *and* b *are units.*

   (2) *If* $ab = 1$ *then* $ba = 1$.

2. §1.4, #25. *Show that every even permutation is a product of 3-cycles.* 8 marks

3. *Let* H *and* K *be subgroups of a group* G, *and define* $HK = \{hk \mid h \in H, k \in K\}$. *If* $HK \subseteq KH$, *show that* HK *is a subgroup of* G. 8 marks

4. (a) *Find the order of* $\bar{2}$ *in* $\mathbb{Z}_{13}^*$. 4 marks

   (b) *Find the order of* $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 8 & 9 & 7 & 6 & 3 & 4 & 1 & 5 \end{pmatrix}$ *in* S_9. 4 marks

5. *Let* a *be an element of order* $|a| = n$ *in a group* G. *Given any integer* m, *let* $d = gcd(n, m)$. *Show that* $\langle a^m \rangle = \langle a^d \rangle$. 8 marks