Fall 2008 PMAT 415 L0 test 1 11:00. 22.09.2008

time: 30minutes

NO CALCULATORS

ID NUMBER: Solutions to Lab Test 1

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Name:	ID:

Problem 1: [5 pts] Assume that $\mathcal{P}A = \mathcal{P}B$. Prove that $A = B$.

Proof. We have to show that $x \in B$ for every $x \in A$ and $x \in A$ for every $x \in B$. The assertion in the problem is symmetric in A and B. Hence it is sufficient to show that $x \in A$ for every $x \in B$.

Let $x \in B$.

Then $\{x\} \subseteq B$ and hence $\{x\} \in \mathcal{P}B = \mathcal{P}A$ implying that $\{x\} \in \mathcal{P}A$. This in turn implies that $\{x\} \subseteq A$ and therefore that $x \in A$.

$\square$

Of course there are several other ways proving the statement. For example, show first that $\bigcup \mathcal{P}(A) = A$.

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Problem 2: [5 pts] Let $\mathcal{A}$ be a set of functions such that for any f and g in $\mathcal{A}$, either $f \subseteq g$ or $g \subseteq f$. Show that $\bigcup \mathcal{A}$ is a function.

Proof. Every function $f \in \mathcal{A}$ is a set of ordered pairs and hence $\bigcup \mathcal{A}$ is a set of ordered pairs, that is a relation. In order to show that the relation $\bigcup \mathcal{A}$ is a function we have to prove that for all $x \in \operatorname{dom} \bigcup \mathcal{A}$ and $y, z \in \operatorname{fld} \bigcup \mathcal{A}$:

$$\langle x, y \rangle \in \bigcup \mathcal{A} \ \& \ \langle x, z \rangle \in \bigcup \mathcal{A} \quad \text{implies} \quad z = y.$$

Let $\langle x, y \rangle \in \bigcup \mathcal{A}$ & $\langle x, z \rangle \in \bigcup \mathcal{A}$.
Then there exists $f \in \mathcal{A}$ so that $\langle x, y \rangle \in f$, that is $f(x) = y$
and there exists $g \in \mathcal{A}$ so that $\langle x, z \rangle \in g$, that is $g(x) = z$.

We may assume without loss of generality that $f \subseteq g$. Which implies that $\langle x, y \rangle \in g$, that is $g(x) = y$.

Hence, $g(x) = y$ and $g(x) = z$ implying that $y = z$ because g is a function. $\square$