Fall 2008 PMAT 415 L0 test 2 14:00. 07.10.2008

time: 30minutes

NO CALCULATORS

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Problem 1: [5 pts] Let f be a one-to-one function from A into A with $c \in A - \operatorname{ran} f$. Define the function $h : \omega \to A$ by recursion so that $h(0) = c$ and $h(n^+) = f(h(n))$. Show that h is one-to-one.

Let N be the subset of elements $a \in \omega$ so that h is one-to-one on a, that is $h(a) = h(b)$ implies $a = b$.

Let $a \in \omega$ with $h(a) = c$. If $a \neq 0$ then there is $n \in \omega$ with $n^+ = a$. Hence $h(a) = h(n^+) = f(h(n)) \in \operatorname{ran} f$ and therefore $h(a) \neq c$ because $c \notin \operatorname{ran} f$. That is $0 \in N$.

We have seen above that $0 \in N$. Let $a \in N$. We have to show that $a^+ \in N$.

Asume for a contradiction that $h(a^+) = h(n)$. It follows from $a^+ \neq 0$ and $0 \in N$ that $h(a^+) \neq c$ and hence that $n \neq 0$ because $h(0) = c$. Hence there is $b \in \omega$ with $b^+ = n$. Then

$$f(h(a)) = h(a^+) = h(n) = h(b^+) = f(h(b))$$

Because f is one-to-one we obtaing $h(a) = h(b)$ and from $a \in N$ that $a = b$ which in turn implies that $a^+ = b^+ = n$. Hence $a^+ \in N$.

We proved that N is an inductive subset of ω and hence equal to ω.

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Problem 2: [5 pts] Let $m, n \in \omega$ with $m \cdot n = 0$. Show that $m = 0$ or $n = 0$.

We prove first that if $0 \neq m \in \omega$ and $x \in \omega$ then $x + m \neq 0$. Let $m \neq 0$ and $N = \{x \in \omega \mid x + m \neq 0\}$. Then $0 \in N$ because $0 + m = m \neq 0$. Let $x \in N$. Then $x^+ + m = (x + m)^+ \neq 0$ because $x + m \in (x + m)^+$. It follows that N is an inductive subset of ω and hence $N = \omega$.

We will prove that if $m \neq 0$ and $n \neq 0$ then $m \cdot n \neq 0$. Let $0 \neq m \in \omega$.

Let $N = \{0\} \cup \{n \in \omega \mid m \cdot n \neq 0\}$. Then clearly $0 \in N$. Also $m \cdot 0^+ = m \cdot 0 + m = 0 + m = m \neq 0\}$.
Let $0 \neq n \in N$. Then

$$m \cdot n^+ = m \cdot n + m \neq 0.$$

Hence N is inductive and therefore $N = \omega$.