Fall 2008 PMAT 415 L0 test 3 14:00. 21.10.2008

time: 30minutes

NO CALCULATORS

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Problem 1: [5 pts] Prove that if $m \in \omega$ and $0 < d \in \omega$, then there exist numbers q and r such that $m = d \cdot q + r$ and $r < d$.

Note that $d \cdot (m+1) \geq 1 \cdot (m+1) = m + 1 > m$, hence $X = \{x \in \omega \mid d \cdot x > m\} \neq \emptyset$.
Let a be the minimum of X and $q = a - 1$, hence $d \cdot q \leq m$. We obtained:

$$d \cdot q \leq m < d \cdot a = d \cdot (q+1) = d \cdot q + d.$$

Let $r = m - d \cdot q$, then:

$$0 = d \cdot q - d \cdot q \leq r = m - d \cdot q < (d \cdot q + d) - d \cdot q = d.$$

It follows that $m = d \cdot q + r$ with $r < d$ and $r \in \omega$ because $r \geq 0$.

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Problem 2: [5 pts] Let A be a nonempty subset of ω such that $\bigcup A = A$. Show that $A = \omega$.

If A contains an element $x \neq 0$ then $0 \in x \in A$ and hence $0 \in \bigcup A = A$.
Otherwise, because $A \neq \emptyset$, we have that $A = \{0\}$, hence $0 \in A$.

We still have to show that $n \in A$ implies $n^+ \in A$ to conclude that A is an inductive set.

$n \in A$ implies $n \in \bigcup A$ implies that there is $m \in A$ with $n \in m$ that is $n < m$, hence $n^+ \leq m$.

If $n^+ < m$ then $n^+ \in m \in A$ and hence $n^+ \in \bigcup A = A$.
If $m = n^+$ then $n^+ \in A$.