



Faculty of Science
Dept. of Mathematics & Statistics

Honours Linear Algebra PMAT 415 - Winter 2011 Homework #1

Due Monday January 24, 2011
(Not all questions will be marked)

1. Show that $\emptyset \neq \{\emptyset\}$.
2. (a) Can you find a set $X \neq \emptyset$ such that for every $x \in X$, x is a subset of X ?
(b) Can you find an infinite such X ?
3. Show that $X \subseteq Y$ if and only if $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$.
4. Express formally (using variables, symbols, conjunctions, quantifiers, ..) the notion that $x \subseteq y$ but $x \neq y$. This is typically abbreviated $x \subset y$, or even $x \subsetneq y$.
5. Let $\epsilon > 0$ and $\alpha > 0$ be rational numbers. Show that there is an $n \in \mathbb{N}$ such that $n\epsilon > \alpha$.
6. Show that any nonempty subset of \mathbb{Z} closed under subtraction is the set of multiples of a given natural number n .
7. (a) Write the elements of $((x, y), z)$.
(b) Can we ever have $((x, y), z) = (x, (y, z))$?
8. Below, f denotes a function.
 - (a) How many functions are there from a set of n elements into a set of m elements?
 - (b) What can you say about the relationship between $f(A \setminus B)$ and $f(A) \setminus f(B)$?
 - (c) What about if in the previous question f is a bijection?
9. For each of the following binary operations determine whether associativity and commutativity hold and determine the existence of a right or left identity element, and in which case the existence of right or left inverse.
 - (a) $X = \mathbb{R}$ and $x \star y = x - y$.
 - (b) $X = \mathbb{R}$ and $x \star y = \max\{x, y\}$.
 - (c) $X = \mathbb{R}$, $a \in \mathbb{R}$ is fixed, and $x \star y = x + y - a$.
10. Let X be a set. The set of functions from $\{0, 1\}$ into X is denoted by 2X . Show that there is a bijection between the sets $X \times X$ and 2X .