UNIVERSITY OF CALGARY

Faculty of Science
Dept. of Mathematics & Statistics

Honours Linear Algebra
PMAT 415 - Winter 2011
Homework #2

Due Monday February 7, 2011
(Not all questions will be marked)

1. Let $E = \varnothing$ and $X = \{\varnothing, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}\}\}$.
   (a) Draw the graph of $(X, \in)$.
   (b) Find all ϵ-minimal elements.
2. Let $X = \{a, b, c\}$.
   (a) Draw the graph of $(\wp(X) \smallsetminus \{\varnothing\}, \subset)$. (Note: $\subset$ means strict inclusion.)
   (b) Find all $\subset$-minimal elements.
3. Show that from the Axiom of Foundation, no set contains itself as an element.
4. For $n \in \mathbb{N}$, show that there are exactly 2^n subsets of a set of n elements.
5. Show that for any property p, there are $x \in y$ such that either $p(x) \wedge p(y)$, or $\neg p(x) \wedge \neg p(y)$.
6. Show that defining ordered pairs as $(x, y) = \{x, \{x, y\}\}$ works, that is
$$(x, y) = (x', y') \text{ if and only if } x = x' \text{ and } y = y'.$$
7. Imagine that the universe of mathematics V was the (good old) natural numbers $V = \{0, 1, 2, 3, ...\}$ and let's define membership by $n \bar{\in} m$ exactly when $n < m$ (let's use $\bar{\in}$ instead of $\in$ so we don't get too confused.)
   (a) Does $(V, \bar{\in})$ satisfy the Axiom of Foundation? Explain.
   (b) Does $(V, \bar{\in})$ satisfy the Axiom of Extensionality? Explain.
   (c) Is there an "empty" set? Explain.
   (d) Does $(V, \bar{\in})$ satisfy the Comprehension Axiom? Explain.
   (e) Does $(V, \bar{\in})$ satisfy the Pairing Axiom? Explain.
   (f) Does $(V, \bar{\in})$ satisfy the Union Axiom? Explain.
   (g) Does $(V, \bar{\in})$ satisfy the Replacement Axiom? Explain.
   (h) Does $(V, \bar{\in})$ satisfy the Axiom of Infinity? Explain.
8. (Bonus) Repeat the previous exercise using $V = \wp(\mathbb{N})$ and $x \bar{\in} y$ exactly when $x \subset y$.