

PMAT 421

MIDTERM

[8] 1. Find the following in the form $x + iy$, simplified as much as possible:

(a) $\cos i$; (b) $\text{Log}(-e)$.

[9] 2. Show that the function $u(x, y) = x^2 - y^2 - 2x$ is harmonic, and find all harmonic conjugates $v(x, y)$. Then write $f(z) = u(x, y) + iv(x, y)$ as a function of z .

[6] 3. Use the Cauchy-Riemann equations to prove that the function $f(z) = \bar{z} + 3$ is not differentiable at any point $z \in \mathbf{C}$.

[12] 4. Evaluate the following contour integrals. You may use any methods, but explain your answers.

(a) $\int_{\mathcal{C}} z |z|^2 dz$ where \mathcal{C} is the straight line from 1 to $1 + 2i$.

(b) $\int_{\mathcal{C}} (z^2 + 1) dz$ where \mathcal{C} is the circle $|z| = 4$.

(c) $\int_{\mathcal{C}} \frac{e^z + 1}{z} dz$ where \mathcal{C} is the circle $|z| = 1$.

[5] 5. Suppose that $f : \mathcal{C} \rightarrow \mathcal{C}$ is a function, and $\alpha, L \in \mathcal{C}$, so that $\lim_{z \rightarrow \alpha} f(z) = L$. Prove, using the $\epsilon - \delta$ definition of limit, that

$$\lim_{z \rightarrow \alpha} 4f(z) = 4L.$$