

Pmat 421 Winter 08
Assignment # 1 due by Monday, Jan 28 by 4 pm

Each questions is worth 5 points.

1. Express in the form $a + ib$, with a, b real: $\left(\frac{i}{1-i} + \frac{1-i}{i}\right)^3$.
2. Find all z (i.e. conditions /restriction on z) for which $(\bar{z})^2 = -|z|^2$.
3. Sketch/Describe the set (a) $|z + i| \leq 2$; (b) $z^2 + (\bar{z})^2 = 2$.
4. Express in the form $a + ib$, with a, b real: $(-1 - i)^8 (1 + i\sqrt{3})^4$, use polar form first.
5. For $z = \left(\frac{1+i}{1-i}\right)^3$ find
(a) (principal branch) $\text{Arg } z$; then (b) find both roots \sqrt{z} .
6. Find all $z \neq 0$ for which $\text{Arg} \frac{1}{z} = -\text{Arg}(z)$.
Explain why it is NOT always true.
7. Show that $|z + w| \leq |z| + |w|$ if and only if $\arg z = \arg w$.
You may use geometry.
8. Find all fourth roots of -1 in the form $a + ib$, with a, b real.
Sketch them on the unit circle.
9. Use De Moivre's Theorem to express $\cos(4\theta)$ in terms of $\cos \theta$ and $\sin \theta$.
10. Sketch the set $\{\text{Im}(z^2) > 1\}$. Is the set open, closed, bounded, connected?
EXPLAIN.