

**PMAT 421      WINTER 00**  
**FINAL            3 hours**

**Name:**

**I.D.#**

1. Find all values (a) of  $(-1)^{1-i}$  (b) of  $\sin(i - \pi)$   
in the form  $a + ib$  where  $a, b$  are real numbers. [6]
2. Find all solutions of  $\sin z = -i$  in the form  $a + ib$  where  $a, b$  are real numbers. [7]
3. Find all  $z$  for which  $\log z = -\log \frac{1}{z}$  if  
(a)  $\log w = \text{Log } w$  principal branch;  
(b)  $\log w$  is the branch where  $\arg w \in [0, 2\pi)$ . [7]
4. Find the Laurent series of  $f(z) = \frac{z}{z+4}$  around  $z_0 = i$   
in the domain containing the point  $10$ .  
Find  $b_2$  and the domain where is the series convergent. [9]
5. Is  $|\sin z| \leq 1$  for all complex  $z$ ? Explain. State the theorem used. [6]
6. Evaluate  $I = \int_c \frac{1}{\sqrt{z}} dz$  where  $c$  is the curve from  $-i$  to  $1+i$  not crossing  
the principal branch cut of the square root function. ( $I = a + ib, a, b$  real) [9]
7. For  $f(z) = \frac{1}{z} e^{\frac{z^2+2}{z}}$   
(a) classify all singularities; (b) find the residue at  $z_0 = 0$ . [9]
8. Evaluate  $\int_0^\infty \frac{\cos \frac{\pi}{4} x}{x^4 - 16} dx$  by means of Residue Theorem. Explain all your steps. [9]
9. Evaluate  $\int_0^{2\pi} \frac{\sin 3\theta}{5 - 3 \sin \theta} d\theta$  by means of Residue Theorem. Explain all your steps. [9]
10. For  $w = z - \frac{1}{z}$  find  
(a) where the mapping is conformal;  
(b) the image of the circle  $|z| = 2$ ;  
(c) the image of the  $y$ -axis minus the origin;  
(d) the image of the unit circle in the  $w$  plane. [9]