Pmat 421
Assignment # 1

Each questions is worth 5 points.

1. Express in the form $a + ib$, with a, b real: $\left(\dfrac{2+i}{3+4i} + \dfrac{2i}{3-4i} \right)^2$.

2. Find when $z\bar{w} = \bar{z}w$ i.e. conditions /restriction on z, w.

3. Sketch the set $|z + i| \leq |z + 2|$. Is it open? Explain.

4. Find all accumulation (limit) points of the set $\{(\operatorname{Re} z)^2 > 1\}$.
   Sketch the set.

5. Express in the form $a + ib$, with a, b real: $(-1 + i)^8 \left(1 - i\sqrt{3}\right)^5$,
   use polar form first.

6. Find (principal branch) Arg of $\left(\dfrac{1+i}{1-i} \right)^3 - 2i$,

   then find both $\sqrt{\left(\dfrac{1+i}{1-i} \right)^3 - 2i}$.

7. Find all z for which $Arg\left(\bar{z}\right) = -Arg\left(z\right)$.
   Explain why $Arg\left(zw\right) = Arg\left(z\right) + Arg\left(w\right)$ is NOT always true.

8. Show that $|z + w| = |z| + |w|$ if and only if $\arg z = \arg w$.
   You may use geometry.

9. Find all fifth roots of -1 in the form $a + ib$, with a, b real.
   Sketch them on the unit circle.

10. Use De Moivre's Theorem to express $\sin(5\theta)$ in terms of $\cos\theta$ and $\sin\theta$.