PMAT 421
Practice Midterm 2

1. Evaluate $\int\limits_c \bar{z}\, dz$ where the curve c is a part of $|z-i|=2$ from $3i$ to $-2+i$.

2. Evaluate $\int\limits_c \frac{e^z}{z^2+4} dz$, where c is positively oriented circle

   (a) $|z-i|=2$; (b) $|z|=3$.

3. Evaluate $\int\limits_c \frac{2z+1}{z^2-z}\, dz$ where

   (a) c is the line segment from i to 2;

   (b) c is the circle $|z|=\frac{1}{2}$.

4. Evaluate $\oint\limits_{|z|=2} \frac{\cos z}{z^3}\, dz$. Explain.

5. Evaluate $\int\limits_c \frac{Log\ z}{z} dz$, where c is

   (a) the line segment from 1 to $1-i$; (b) $|z|=1$.

6. Evaluate

   $\int\limits_c z\,|z-1|\, dz$ where the curve c is a part of $|z-1|=5$ where $\operatorname{Im} z \geq 0.\backslash$

7. Estimate $\int\limits_c \frac{e^{iz}}{\bar{z}} dz$ if the curve c is

   (a) half of a circle $|z|=4$ from $-4i$ to $4i$;

   (b) the line segment from -3 to $4i$.

8. Show that $\left|\int\limits_c e^{\bar{z}\operatorname{Im} z} dz\right| \leq 2\pi e^{\frac{R^2}{2}} R$ where c is the circle $|z|=R$ positively oriented.

   HINT:You may use Langrange multipliers method to find M.

9. Evaluate $\int\limits_c z^{-\frac{1}{2}}\, dz$ where $z^{-\frac{1}{2}}$ is the branch where $\arg z \in [0, 2\pi)$

   and c is a curve connecting $-1-i$ to i,not crossing the branch cut.

10. Find the radius of convergence of $\sum\limits_{n=1}^{\infty} n! \left(\frac{z}{n}\right)^n$.

11. Find a Laurent series around $z_0 = -1$ of $f(z) = \dfrac{1}{z^2 + z}$ in a domain containing the point $3i$.Describe the domain.

12. Find the condition on z such that $\sum_{n=1}^{\infty} \left(\dfrac{1}{z^2 + 1} \right)^n$ is convergent.Find the sum .

13. For $f(z) = \dfrac{z}{z^2 + z - 2}$

    (a) find the Laurent series around $z_0 = 1$ particularly b_1 and a_9, in a deleted neighborhood of 1. Where is the series convergent?

    (b) find the Laurent series around $z_0 = 0$ in the domain $1 < |z| < 2$.