

PMAT 421 WINTER 00
FINAL 3 hours

1. Find all values (a) of $(-1)^{1-i}$ (b) of $\sin(i - \pi)$
in the form $a + ib$ where a, b are real numbers.
2. Find all solutions of $\sin z = -i$ in the form $a + ib$ where a, b are real numbers.
3. Find all z for which $\log z = -\log \frac{1}{z}$ if
 - (a) $\log w = \text{Log } w$ principal branch;
 - (b) $\log w$ is the branch where $\arg w \in [0, 2\pi)$.
4. Find the Laurent series of $f(z) = \frac{z}{z+4}$ around $z_0 = i$
in the domain containing the point 10 .
Find b_2 and the domain where is the series convergent.
5. Is $|\sin z| \leq 1$ for all complex z ? Explain. State the theorem used.
6. Evaluate $I = \int_c \frac{1}{\sqrt{z}} dz$ where c is the curve from $-i$ to $1+i$ not crossing
the principal branch cut of the square root function. ($I = a + ib, a, b$ real)
7. For $f(z) = \frac{1}{z} e^{\frac{z^2+2}{z}}$
 - (a) classify all singularities; (b) find the residue at $z_0 = 0$.
8. Evaluate $\int_0^\infty \frac{\cos \frac{\pi}{4}x}{x^4 - 16} dx$ by means of Residue Theorem. Explain all your steps.
9. Evaluate $\int_0^{2\pi} \frac{\sin 3\theta}{5 - 3 \sin \theta} d\theta$ by means of Residue Theorem. Explain all your steps.
10. For $w = z - \frac{1}{z}$ find
 - (a) where the mapping is conformal;
 - (b) the image of the circle $|z| = 2$;
 - (c) the image of the y -axis minus the origin;
 - (d) the image of the unit circle in the w plane.