Pmat 421 W 11
Assignment # 1 due by Mo,Jan. 24,2011

Each questions is worth 5 points.

1. Express z in the form $a+ib$, with a,b real where $z=\left(\frac{3+i}{2-i}+\frac{2+3i}{3-2i}\right)^2$.

2. Describe/sketch the following sets:
   a) $\mathrm{Re}(z+i)=5$;**b)** $\mathrm{Im}(z+i)=5$; .**c)** $|z+i|=5$.

3. Find all z for which $z^2=1+2i$.

4. For $z=(-1+i)^6(\sqrt{3}-i)^5$ find (principal branch) $Arg\ z$.
   .

5. From the definition of absolute value $|...|$ for complex numbers
   show that $\left|z^2\right|=|z|^2$ for any complex z and $\left|\frac{1}{z}\right|=\frac{1}{|z|}$ for $z\neq 0$,
   then prove $|z^n|=|z|^n$ for any integer n.Justify each step.

6. Describe the set of all z such that $|\pi-Argz|<\frac{\pi}{4}$.

7. Find all roots $\left(-8-i8\sqrt{3}\right)^{\frac{1}{4}}$ in the from of $a+ib$, with a,b real.

8. Use De Moivre Theorem to express $\cos(5\theta)$ in terms of $\sin\theta$ and $\cos\theta$.

9. Prove the formula $2\sin x\sin y=\cos(x-y)-\cos(x+y)$ for any real x,y using the properties of complex function e^{ix} and $\sin x=\mathrm{Im}\,e^{ix},\cos x=\mathrm{Re}\,e^{ix}$.

10. Describe the set $S=\{z;z\bar{z}\geq|z|\}$ .Is it open,closed,bounded,connected?
    Is $z=0$ an interior, exterior, boundary or accumulation point of S ?
    Explain.