Pmat 421 W09 Assignment # 1 due by Jan. 23,2009

Each questions is worth 5 points.

- 1. Express z in the form a+ib, with a, b real where $z = \frac{(1+i)(3-i)(-2-i)}{i(3+4i)(5-i)}$.
- 2. Find the sum $\sum_{n=0}^{300} i^n$.

3. Express z in the form a + ib, with a, b real where $z = \frac{(-1-i)^3}{(-\sqrt{3}+i)^2}$ use the polar from of -1 - i and $-\sqrt{3} + i$. Find (principal branch) Arg of z.

4. Show that $|z + w|^2 + |z - w|^2 = 2|z|^2 + 2|w|^2$ for any complex z and w.

5. Solve $z^{\frac{4}{3}} + 2i = 0$ i.e. find all possible z in the form a + ib, a, b real.

- 6. Find all z for which (a) $\operatorname{Im} \frac{1}{z} = -\operatorname{Im} z$ (b) $\operatorname{Re} \frac{1}{z} = -\operatorname{Re} z$.
- 7. Find all roots $(-8)^{\frac{1}{3}}$.
- 8. Define the branch of the argument where $\arg z = \theta \in [0, 2\pi)$ in terms of x and y for any $z = x + iy, z \neq 0$

For which z the following is true $\arg \bar{z} = -\arg z$ (the branch from above).

9. Find all accumulation points of the set $\{(-i)^n; n = 1, 2, 3, ...\}$.

10. Describe or sketch the set $\left\{z; \operatorname{Im} \frac{1}{z} \geq \frac{1}{6}\right\}$. Is the set open, closed, bounded, connected? Explain.