

time: 50 minutes

ID NUMBER:

NO CALCULATORS

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**Problem 1:** [56 pts] Define:

1. The sequence  $\{a_n\}$  converges to the real number  $A$ .
2. The set  $E \subseteq \mathbb{R}$  is open.
3. The sequence  $\{a_n\}$  is eventually constant.
4. The sequence  $\{a_n\}$  is a Cauchy sequence.
5. The function  $f : D \rightarrow \mathbb{R}$  is uniformly continuous.
6. Accumulation point of a subset of  $\mathbb{R}$ .
7. The function  $f$  has limit  $L$  as  $x$  tends to  $a \in \mathbb{R}$ .
8. The function  $f$  is continuous at  $a \in \mathbb{R}$ .

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**Problem 2:** [54 pts]

1. Define the function  $\exp(x)$ .

2. Prove that  $\lim_{x \rightarrow 0} \exp(x) = 1$ .

3. Prove that the function  $\exp(x)$  is continuous.

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**Problem 3:** [55 pts] Let  $D$  be a closed and bounded set. Prove: If  $f : D \rightarrow \mathbb{R}$  is continuous then  $f$  is uniformly continuous.

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**Problem 4:** [55 pts] Prove: If a function  $f$  is continuous on  $[a, b]$  and if  $f(a) < k < f(b)$  then there exists a real number  $c \in (a, b)$  such that  $f(x) = k$ .