PMAT 505 ASSIGNMENT 4 Due November 22, 2010

1. Munkres p.144, 2. [20]

2. Answer each of the following "True" or "False." If False, provide a counterexample. Note that all counterexamples, except for one, can be found using suitable subspaces of $\mathbb{R}$. [20]

   (i) $\mathrm{Int}(A \cup B) = \mathrm{Int}(A) \cup \mathrm{Int}(B)$ ______

   (ii) $\mathrm{Int}(A \cap B) = \mathrm{Int}(A) \cap \mathrm{Int}(B)$ ______

   (iii) $\mathrm{Int}(\mathrm{Int}(A)) = \mathrm{Int}(A)$ ______

   (iv) $\mathrm{Bd}(\mathrm{Bd}(A)) = \mathrm{Bd}(A)$ ______

   (v) Any quotient space of a T_1 space is T_1 ______

   (vi) Any quotient space of a metric space is T_2 ______

   (vii) Let Top_q have as objects all topological spaces, and as morphisms all quotient maps. Then Top_q is a category. ______

   (viii) If $(X, \mathcal{T})$ is T_2 and $\mathcal{T}'$ is another topology for X with $\mathcal{T} \subseteq \mathcal{T}'$, then $(X, \mathcal{T}')$ is also T_2. ______

   (ix) An infinite product of discrete spaces (with the product topology) is also discrete. ______

   (x) The space $\mathbb{R}_l$ is T_3. ______

3. (a) In the category Top, prove that any epimorphism is surjective and any monomorphism is injective.

   (b) In the category $Haus$, show that an epimorphism need not be surjective. [20]

4. Prove that $|\hom_{Top}(\mathbb{R}, \mathbb{R})| = \mathbf{c}$, i.e. the cardinality of all continuous functions from the reals to the reals equals $\mathbf{c}$, the cardinality of the continuum.

   [10]

5. State and prove the dual result to Theorem 22.2 for inclusion maps.

   [10]

6. Munkres, p.171, 4. [20]