

Pure Mathematics 511/ 611

Algebra III/Algebra IV

(see Course Descriptions under the year applicable: <http://www.ucalgary.ca/pubs/calendar/> )

### Syllabus

<u>Topics</u>	<u>Number of hours</u>
Recap of rings and ideals; isomorphism theorem	3
Integral domains; maximal and prime ideals; principal ideal domains (PIDs)	3
The ascending chain condition; unique factorization in PIDs	3
Modules over rings; submodules; quotient modules; linear transformations and kernels	3
Direct sums of modules; free modules; basis and rank; the matrix of a linear transformation	3
Tensor product; extension of scalars	3
Symmetric and alternating products; determinants	3
Cyclic and torsion modules over PIDs	3
The structure of finite abelian groups; canonical forms of matrices	3
Finitely generated modules over PIDs; invariant factors	3
Presentations of modules; computing invariant factors	3
Exact sequences; hom and tensor functors and their adjointness, left and right exactness	3
Projective, injective and flat modules (time permitting)	
<b>TOTAL HOURS</b>	<b>36</b>

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# PMAT 511 Abstract Algebra III

## Course Outcomes

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### General outcomes

By the end of this course, students will be fluent in several central and advanced techniques of Algebra.

### Specific outcomes

Specifically, by the end of this course, students will be fluent in the following three topics:

- The theory of finite fields
- The structure theorem for finitely generated modules over a principal ideal domain, and
- Applications to the proof of the rational and Jordan canonical forms for matrices over a field.

Moreover the students will be fluent in aspects of one or more of the following topics:

1. Projective and injective modules, the injective hull, projective dimension, the functors  $\text{hom}(M;N)$  and  $M \otimes N$ .
2. The Wedderburn-Artin structure theorem for semiprime rings with the descending chain condition for left ideals.
3. The Lasker-Noether theory of commutative rings with the ascending chain condition on ideals.
4. Introduction to commutative algebra.
5. The theory of solvable and nilpotent groups.
6. An introduction to Galois Theory for finite field extensions.

### A student who successfully completes this course will:

7. Have a global appreciation of these algebraic systems.
8. Understand the use of the basic theorems about these systems and how they shape the development of the system.
9. Be able to see some of the interconnections between the systems under study and related systems.