

Take-Home Problems, PMAT 519

Due April 28, 2004. Partial solutions can be submitted.

- 1 Working over $GF(7)$ let G be the cyclic R.S. code with generator polynomial $(x-1)(x-2)(x-4)$.
 - (a) Write down the dual code (= parity-check code) H for G as developed in class; H is also a R.S. code.
 - (b) Write down H using the polynomial $h(x)$ where $g(x)h(x) = x^6 - 1$
 - (c) How can the different-looking answers be reconciled?
 - (d) What is the minimum distance of H ?
- 2 Briefly show that the dual of an M.D.S. code is also M.D.S.
- 3 Give an example of a cyclic linear code C with $C = C^\perp$.
- 4 Show that for a prefix encoding of a source with N source words we have that $l_1 + l_2 + \dots + l_N \geq N \log N$ where l_1, l_2, \dots, l_N are the lengths of the code words. (As usual we assume that the source is memoryless)
- 5 Show that, for the Huffman algorithm, $l_1 + l_2 + \dots + l_N \leq \frac{1}{2}(N^2 + N - 2)$.
- 6 A survey of I.T. professionals reveals that 40% are wealthy, 70% own a car and 90% of wealthy students own a car. How much information about the ~~richness~~ wealth of an individual is revealed by whether or not the individual owns a car?
- 7 If the average length of a codeword in the encoding of a memoryless source is at least $N > 1$ and the code is prefix, must the source have at least 2^N words?